Coordinating in the haircut

A model of sovereign debt restructuring in secondary markets

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Abstract

Over the last two decades, countries that default on their debts increasingly have had to confront mostly atomistic unconnected bondholders when engaging on restructuring negotiations. According to data, the nature of creditors impacts on restructuring results, reducing investors concession to the government in default. This paper, proposes a model to study the determination of the haircut for defaulted debt when bondholders play a coordination game. The Resulting multiplicity is solved with a global games approach. I find that this new market setting introduces an additional constraint to the government which end up compressing the asked concession in order to increase the probability of program's acceptance. For illustrative purposes I run simulations with calibrated parameters and find that coordination costs account for a significant portion of the haircut reduction (up to 25%) after sovereign debt disintermediation process.

Keywords: Sovereign debt; Restructuring; Secondary market; Coordination; Global games.

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1 Introduction

The disintermediation of government debt in emerging economies after 1980 (Andritzky (2006), Brum and Della Mea (2012), Das et al. (2012)) embedded a coordination feature into many debt related processes. In some of them, such as the issuance and pricing of debt, the market helped to avoid coordination frictions. In the case of restructuring after a default, on the contrary, a decision stage with no public information available, turned bondholders' coordination a critical hurdle in the process. In this paper, I propose a model to determine the outcome of the restructuring process when the defaulted debt is entirely composed of bonds traded at capital markets. The solution consists of an equilibrium in the strategic interaction between a constrained sovereign and a continuum of bondholders playing a coordination game of strategic complements.

Three milestones paved the way for the transition of debt from loans to bonds in emerging economies (Andritzky, 2006). First, the establishment of a high yield market to host the trading of this risky debt. Second, the Brady Plan in 1989 securitized government defaulted bank loans into bonds traded at the novel market. And third, the liberalization of capital markets which fueled investment flows into assets abroad. This process attained considerable proportions in only two decades. For instance, the contribution of bank loans, replaced by bonds, to the private stocks of emerging sovereign debt plummeted from 80 per cent to 26 per cent (figure 1a). At the same time, the proportion of bonds subject to default and restructuring experienced a sharp increase (figure 1b).

Although the new setting seems to weaken creditors' position, the unexpected consequence was the worsening of government payoffs in debt restructurings. Indeed, an inspection on Cruces and Trebesch (2013) database[1] suggests that, on average, the concession to the government in restructurings reduced from 66% to 39% after the transition from bank loans to bonds.[2] Moreover, regression analysis on this data cannot reject a negative relationship between the type of creditor and the restructuring outcome (see Appendix A). Similarly, Bai and Zhang (2019) on Benjamin and Wright (2009) database find that the change in the sovereign debt setting reduced government's outcome by 14%. In their paper, the cause is a contraction in total bargaining time as capital markets reveal creditors' outside option previously kept as private information. In contrast, I rather simplify the time dimension and focus on the effects of the alignment of strategies of uncoordinated investors, finding that it entails a cost for the government in the restructuring outcome.

This paper analyses the effects of investors' strategic behavior on government's restructuring outcome or *haircut*.[3] The bondholders deciding whether

[1] The authors updated their database in 2014.

[2] The selected period expands from 1998 to 2014 and avoids restructuring processes inside Brady plan as they might have had special features that might add noise to the comparison.

[3] While renegotiating with creditors the restructuring terms on defaulted debt, the government can offer a plan that includes one or many instruments such as increasing maturities, buybacks, cash tenders, bond exchanges, face value reduction among others. The haircut here

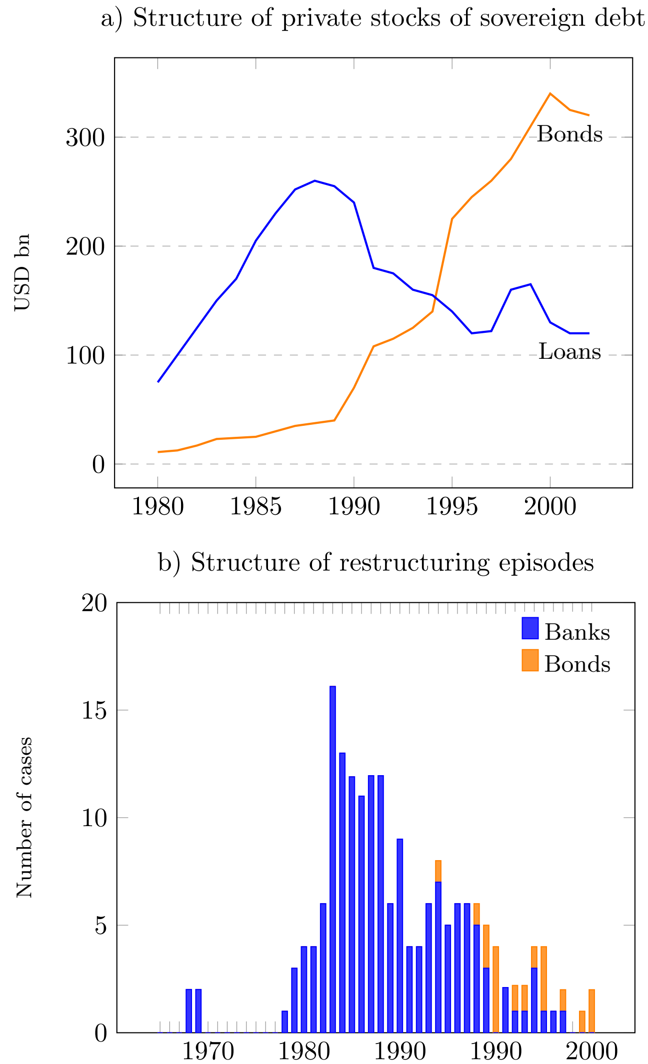

Figure 1: Bonds *versus* loans. (a) Structure of external public debt in emerging market countries, stocks of privately held debt (Borensztein et al., 2004). (b) Finalized restructurings per year (Andritzky, 2006)

they accept or reject government's restructuring proposal play a coordination game of strategic complements in which the highest payoffs are obtained when most players simultaneously align their strategies. In this particular case, acceptance when most reject implies a participation cost (for instance reputational) and rejection when most accept entails a lower payment at the secondary market. But while most agents would then prefer to coincide in their decisions, the process itself makes it difficult to happen. On the one hand, once the proposal has been released there is a window of time for investors to decide in which there is no official information about the total acceptance rate achieved. On the other hand, there are thousands of bondholders[4] and spread enough to make it difficult gathering the necessary amount to secure an agreed position.[5]

This paper proposes a three stages game to solve the restructuring process between the defaulting government and the bondholders. In the first stage, the former releases its restructuring plan. Information regarding the state of the economy at this stage is normalized to zero and only after the proposal has been communicated to the market, the nature draws and reveals new information about an economic fundamental. Such information sequence simplifies the signaling feature of the restructuring process: responses of the market to information embedded in government's announcements.

In the second stage, bondholders confront a coordination game, deciding simultaneously its acceptance of government's proposal. Theoretical models of coordination games bring about multiple Nash equilibria. In this particular case, we have a high payoff equilibrium where most investors accept the proposal, and a low payoff equilibrium in which most reject it. Bondholders aligning into the no participating choice would consist of a coordination failure as that is the less beneficial choice among all (Pareto inferior). In order to predict which equilibrium will be played I introduce global games. Proposed by Carlsson and van Damme (1993), this scheme restricts complete information assumption to solve for one equilibrium using iterative deletion of dominated strategies. In particular, Proposition 1 identifies a unique value in the fundamental set that divides accepting and non accepting bondholders as signal noise converges towards zero. Solving the multiplicity in the coordination game at the second stage, allows us to use backwards induction to find the solution to the complete model from the third to the first stage and thus obtain the optimal haircut that uniquely solves the restructuring process.

Finally, in the third stage, the government observes both the unique equilibrium obtained previously (using the threshold in Proposition 1) expressed as a percentage of total agreement, and the information of the fundamental of the economy, and combine them to decide whether it pays as proposed and change

summarizes the total equivalent percentage loss of the investment in terms of the net present value.

[4] As were the cases of Dominica in 2004, Pakistan in 1999, Uruguay in 2003, Seychelles in 2009 and even hundred of thousands such as the cases of Ukraine in 2000 (100 thousand) or Argentina in 2005 (600 thousand). (Andritzky, 2006).

[5] According to Das et al. (2012) there exist few experiences of successful representative groups in sovereign bond restructuring events.

the state or continues on default to eventually starts a new restructuring afterwards. Proposition 3 expresses the coordinating haircut in terms of model fundamentals and Proposition 4 present the necessary conditions for uniqueness of the solution.

To assess model results, I obtain comparative statics and run two different exercises. In the first one, there is a comparison between the coordination haircut and the haircut widely used in restructuring literature, obtained using Nash bargaining. I find that under certain conditions the Nash bargaining haircut overstates the coordination haircut. The difference increases with the atomization of investors (and the loss of their bargaining power). In the second exercise, I propose a procedure to quantify the government costs of the transition from bank towards market financing, which I will call the *coordination costs*. I find that the coordination feature actually restricts government's negotiating power reducing the haircut. Indeed, coordination costs can reach a significant level in simulations, accounting for almost one fifth of the average difference between haircuts in banks and bonds restructurings. In the model, the inability to negotiate with its counter parties, forces the government to send the market a strong signal in order to convince bondholders both to participate themselves and that others will alike (which are respectively first and second order believes).

This paper relates with the increasing literature that studies different technical aspects of sovereign debt restructuring. For instance Pitchford and Wright (2012) present an n investors bargaining model of alternative offers to study delay as a consequence of holdout and free riding strategies. They use their model to evaluate how contractual innovations can help solving distortions. Benjamin and Wright (2009) also study delays in restructuring processes and find that they are functional to both, investors and the government, as they allow them to increase their payoffs. As mentioned before, Bai and Zhang (2019) use a private information model to analyze how the disintermediation of sovereign debt has reduced the length of the negotiations. they argue that the secondary market replaced the endless bargaining process between sovereign and investors as a method to reveal each other's outside option. All these papers share in common that they assume a representative bondholder negotiating directly with the government (on behalf of the universe of bondholders) through one or multiple rounds. From another perspective, Bi et al. (2016) focus on the embedded coordination problem in the new market setting but they avoid equilibrium multiplicity proposing an adjustment of investors' payoffs. In another section, they use their model to endogenize the haircut now using sunspots to project the equilibrium at the investors' coordination game. This paper aims to contribute to this literature by addressing an unexplored dimension: the cost on the restructuring outcome of the coordination effort. Besides, unlike previous studies, the novel use of global games to project the final equilibrium allows me to solve multiplicity in terms of the observed fundamental which I think allows further to explain the final result.

Carlsson and van Damme (1993) first propose global games to solve multiplicity in coordination games using information constraints. They demonstrate that by introducing some noise into private information, players are forced to estimate others' participation in terms of the distribution of a fundamental then allowing us to solve equilibrium multiplicity by iterative deletion of strictly dominated strategies. This paper constitutes a first application of this technology to solve multiplicity in theoretical models of sovereign debt restructuring. Other applications include financial markets, taxes, business cycles, prices and other public signals.[6]

Finally, this paper also relates to the broad literature on sovereign debt with endogenous default originated in Eaton and Gersovitz (1981) and in particular to the sub set of models containing endogenous restructuring. The first approach is proposed in Yue (2010), obtaining the haircut as an output of the model and applied to replicate Argentina's crisis in 2001. Asonuma and Trebesch (2016) adds to the process the negotiation over the return of the bond exchanged for the defaulted securities, and Dvorkin et al. (2018) a negotiation on the maturity of the new securities. Asonuma and Joo (2019) use one such model to analyze the dynamics of public investment during both default and the restructuring processes. All these quantitative models use Nash bargaining to solve debt renegotiation and, as a consequence, they simplify the n-dimensional feature of the investors and the resulting coordination effects on the outcome. The contribution to these works is a methodology to determine the haircut that takes into account the atomization of the investors set. Indeed, incorporating the coordination feature I find a unique haircut in terms of the fundamental and the specific parameters of the model (recovery at secondary market, participation costs, holdouts provisions). When the *coordination haircut* is compared to the Nash bargaining haircut, I find that both solutions coincide in a scenario where the government possesses complete bargaining power, and differ at the opposite one, where investors own all the bargaining power (for each possible bargaining power of the agents). In such case, besides, the Nash bargaining solution situates above the coordination one, due to the fact that the coordination of investors end up transferring a cost to the government in terms of the haircut.

The paper is organized as follows. Section 2 presents the model. The structure advances through two successive specifications: a complete information approach with multiple equilibria and a global games approach with incomplete information which ends up solving the restructuring by finding the optimal haircut. Then I present the resulting comparative static of the solution in terms of the parameters in the model. In section 3 I add an illustration to estimate possible coordination costs, and a comparison against Nash bargaining haircut. Section 4 concludes and discusses further analysis roads.

2 Model

The model consists of a three stages game in which agents interact strategically to determine the haircut of the defaulted debt. There are two kinds of agents, the government (or sovereign) who defaulted on a set of outstanding debt and a continuum of investors holding one unit of defaulted bond each. Bondhold-

[6] See Taylor and Uhlig (2016) for a detailed survey.

ers' decisions are modeled within a coordination game which entails multiple equilibria with complete information. I then introduce incomplete information à la Morris and Shin (1998) using the global games approach as an equilibrium selection device. This setting allows us to identify a threshold in economic fundamental that determines strict dominance regions ruling investors' decisions, and then an optimal haircut that solves the game.

2.1 The game in the time line

Figure 2 presents the flow of the model. At stage zero, the government defaults on a subset of total outstanding bonds.[7] In the first stage, the government proposes a haircut h, corresponding to the total concession asked to the creditors of defaulted debt. Then nature draws a value for random variable θ which represents sovereign's current payment capacity and reveals it to all the agents. At the second stage, a continuum of creditors with a unit face value bond each, observe both the announced h and the fundamental θ, and determine an individual binary action a_i of acceptance (rejection) of government's proposal. In the last stage, the government knows θ and the acceptance rate (ℓ) and determines the result of the negotiation R. The process ends at this stage in any case. If acceptance rate reaches a required threshold (in line with government's constraint), government pays bondholders and holdouts and exits default; otherwise, it exits negotiation without switching the state. We will solve this game using backwards induction from the last stage to the first one.

2.2 Agents' payoffs

2.2.1 Sovereign

In its final decision stage, the government uses all the available information to evaluate the result of the restructuring proposal. The information set at third stage includes both the economic fundamental θ and the aggregate acceptance rate $\ell \in [0,1]$ for the program. We will denote this set with $\mathcal{I}^3_{gov} = \{\theta, \ell\}$, using super scripts to index the stage and under scripts to index the agent of reference. θ represents sovereign's current payment capacity. It is drawn by nature from a probability density with boundaries $[\check{\theta}, \hat{\theta}] \in \mathbb{R}$. The haircut h represents the total equivalent percentage loss on investment for bondholders (as in Sturzenegger and Zettelmeyer (2008)).

The expression in (1) describes government's normalized payoffs $G(\ell, h, \theta)$ as a function of the acceptance rate, the haircut and the observed payment capacity. In the first line, the government proposed a restructuring plan with haircut h which gathered total acceptance level ℓ. When the restructuring proposal is successful ($R = 1$) the sovereign pays h to participating bondholders as announced and provisions ν for those rejecting the plan.[8] In consequence,

[7] These are plain-vanilla contracts, with no special provisions in the event of default.

[8] Using data from US corporate debt, holdout premium against early settlers situated at 11% in 115 restructurings between 1992 and 2000 (Fridson and Gao, 2002) and at 30% in 202

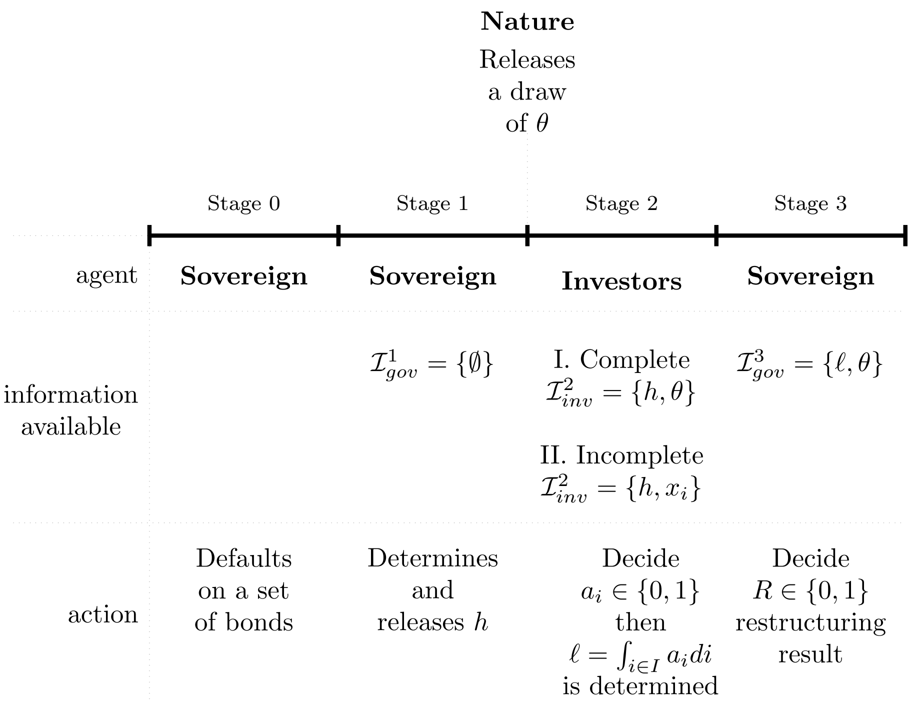

Figure 2: Time line.

it exits default receiving a boost of $\xi \in [0, 1]$ in payment capacity θ.[9] Such impulse could derive from recovering access to capital markets, ease of international sanctions, bailout funds received, implementation of structural reforms, political and financial distress amongst others. In the second line, the proposal gathers a low market acceptance (below government´s required threshold), the renegotiation fails ($R = 0$) and government remains in default state with fundamental θ.

$$G(\ell, h, \theta) = \begin{cases} \theta(1+\xi) - [\ell(1-h) + (1-\ell)\nu] & \text{if } R = 1 \\ \theta & \text{if } R = 0 \end{cases} \tag{1}$$

For the government to engage in a restructuring plan, it is required that $\theta\xi \geq \ell(1 - h - \nu) + \nu$ in (1), which implies that post restructuring assets gain has to exceed the total compromised payments.

Let us assume exogenous holdouts payment provision ν lays inside $(1-h, 1]$.[10] When $\nu = 1$ the government pays the full bond value to the share of holdout investors $1 - \ell$. Were $\nu = 1 - h$, the government would pay the same amount

restructurings between 1980 1992 (Altman and Eberhart, 1994).

[9] Annual median growth in restructuring countries increases from 1.5% previous the final agreement to 4 to 5% after it (Das et al. (2012) on Trebesch (2011) data set).

[10] This total value would eventually be settled by the government or a judge in court.

to all debtors whether they accept or not, so that participation rate would turn irrelevant. In this case, government could announce $h = 1$ and yet exit default without any repayment to bondholders. Thus, let us assume $\nu > 1 - h$, to abstract from this trivial scenario.

Note that when determining the haircut at stage one government's information set is normalized to $\mathcal{I}^1_{gov} = \{\emptyset\}$ forcing it then to use the distribution of θ.

2.2.2 Bondholders

A bondholder i from a continuum set of measure one of investors with one unit of bond each, chooses an individual action $a_i \in \{0, 1\}$, which represents rejection or acceptance of the proposed repayment program. Each bondholder is characterized by a utility function $u(a, \ell, \theta) : \{0, 1\} \times [0, 1] \times [\check{\theta}, \hat{\theta}] \to \mathbb{R}$ in (2).

$$
\begin{aligned}
u(0, \ell, \theta) &= \begin{cases} \delta\nu & \text{if} \quad R = 1 \\ 0 & \text{if} \quad R = 0 \end{cases} \\
u(1, \ell, \theta) &= \begin{cases} 1 - h - m & \text{if} \quad R = 1 \\ -m & \text{if} \quad R = 0 \end{cases}
\end{aligned} \tag{2}
$$

Both the proposal success (assessed by the government in the last stage) and bondholders payoffs will depend on achieving a minimal level of aggregate acceptance ex-post ($\ell > \ell^*$). However, each agent's information set at decision stage is the singleton $\mathcal{I}^2_{in} = \{\theta\}$. As the aggregate acceptance level is unknown while deciding, each agent uses a uniform prior over others' actions (assigns the same probability to each acceptance rate level $\ell \in (0, 1)$).

Rejecting agents ($a_i = 0$ in (2)), expect a recovery value of $\delta\nu$ ($\delta \in [0, 1]$) for unit of bond in a successful proposal and 0 otherwise. Agents accepting ($a_i = 1$ in (2)) receive $1 - h$ when negotiation prospers or 0 otherwise, spending in any case participation costs $m > 0$ (for example to acquire the information).

Note that holdouts payments and receipts do not coincide. Here, $1 - \delta$ is a non participating loss which should be interpreted as litigation expenses, the probability of not receiving that amount or the wait until that happens.[11] Then $\delta\nu$ is the expected recovery value and as such, should coincide with the bid price of the defaulted bond at the junk market.

This utility function may portray the bulk of investors for whom expected gains might not compensate litigation costs (low δ). The opposite case is the small group of professional holdouts[12] that buys the defaulted debt at secondary market and affords many years of litigation with sovereign at international courts with considerable return. I do not exhaustively include them in this model, as these group generally weights less than 10% of total outstanding (Das et al., 2012), and their behavior does not coincide with the mass of bondholders.

[11] Wright (2011) calibrates restructuring costs in its Nash bargaining model as 3.5% of renegotiated debt, from which 90% falls upon lead investor.

[12] Some of them are Water Street, Elliot Associates, Cerberus, Davidson Kempner, Aurelius Capital.

2.3 Full information and multiplicity

In this section I use backwards induction to solve the model starting from the last stage to the first one. Complete information entails multiple equilibria in stage 2 which will derive towards incomplete information to solve the game into a unique equilibrium.

2.3.1 Stage 3

Imposing government indifference condition in (1) we obtain a minimal threshold $\ell^*(\theta)$ for investors' acceptance ℓ in terms of the fundamental. Only for ℓ values above that level, the government pays as agreed and exits default.

$$\ell^*(\theta) = \frac{\nu - \xi\theta}{\nu - (1-h)} \tag{3}$$

Lemma 1. *Government softens the required acceptance level in the case of better economic conditions (higher θ), when it commits to a lower repayment (higher h) or when exiting boost (ξ) is higher. On the contrary, $\ell^*(\theta)$ raises when payments to holdouts ν are higher (if $\theta > \frac{1-h}{\xi}$, and lower otherwise).*

Proof: The cases of ξ, θ and h are trivial.

$$\frac{\partial \ell^*(\theta)}{\partial \nu} = \frac{-(1-h) + \xi\theta}{(\nu - (1-h))^2} \geq 0 \text{ for } \theta \geq \frac{1-h}{\xi} \blacksquare$$

Thus given h, the response to increases in holdout payments is not linear. A compromised government position (θ below $\frac{1-h}{\xi}$) forces it to reduce $\ell^*(\theta)$ each time ν augments with the purpose of gathering additional acceptance to finance the more expensive holdout payments. On the contrary, high fundamental draws allow the government to increase the threshold in response to increases in ν if it estimates the exit might be too expensive.

Evaluating $\ell^*(\theta)$ in the boundaries of $\ell \in [0,1]$ we can identify strict dominance regions in government's strategies as a function of fundamental θ (see figure 3). Thus $\underline{\theta} \equiv \frac{1-h}{\xi}$ and $\bar{\theta} \equiv \frac{\nu}{\xi}$ denote the pair of benchmark values of θ which demands limit levels of engagement, $\ell^*(\underline{\theta}) = 1$ and $\ell^*(\bar{\theta}) = 0$ respectively, to exit default. As a consequence, for $\theta \in [\check{\theta}, \underline{\theta})$ the default is the best response regardless of ℓ due to the extremely reduced payment capacity, while for $\theta \in [\bar{\theta}, \hat{\theta}]$ abundant assets make exiting default the best strategy even at null acceptance level.

Intuitively, using these limits in the utility function (1), low values of θ request each circulating bond to accept the haircut h ($\ell^* = 1$) in order to exit default, and in this case all the assets gain $\xi\theta$ would be applied to comply with the program $(1-h)$. On the contrary, high fundamental draws such as $\bar{\theta}$ can afford every obligation and even the holdouts at the highest rate at which it would apply only the asset gain $\xi\theta$.

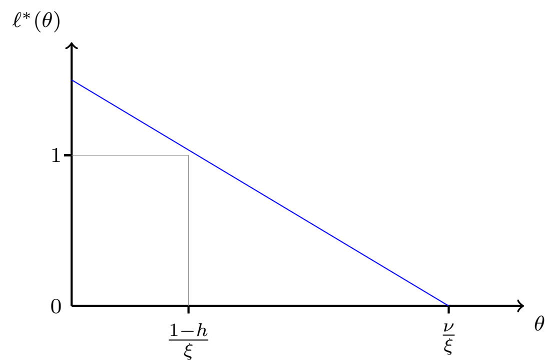

Figure 3: Threshold $\ell^*(\theta)$.

2.3.2 Stage 2

This stage constitutes a symmetric binary action coordination game. I will use a general version with a continuum of agents. The reader can find a full-information-two-investors illustration in appendix B.

With complete information, each value of θ determines a subgame where investors decide their strategy $a_i(\theta)$ as a best response to others' behavior in the corresponding scenario. We then obtain a pair of equilibrium strategies, depending on variable h and on parameters m, δ and ν. Strategy sets are denoted as follows: each element (parenthesis) represents a range for the observed fundamental value separated by the boundaries $\underline{\theta}$ and $\bar{\theta}$; inside parenthesis there are the responses to low and high aggregate acceptance level (against the threshold ℓ^*) respectively. Thus we have two possibilities:

$$\delta\nu > 1 - h - m \qquad a_i(\theta) = \{(0,0),(0,0),(0,0)\}$$

$$\delta\nu < 1 - h - m \qquad a_i(\theta) = \{(0,0),(0,1),(1,1)\}$$

For $\theta \in [\check{\theta}, \underline{\theta})$ sovereign continues on default regardless of the participation rate ℓ. Participation payoff is strictly below non-participation's : $u(1,\ell,\theta) = -m < u(0,\ell,\theta) = 0$. The strictly dominant strategy is to reject proposal if $m > 0$, for any $\delta\nu$ at both possible scenarios of acceptance (low and high), and as a result $\ell = 0$.

For $\theta \in [\underline{\theta}, \bar{\theta})$ sovereign's decision depends on overall acceptance ℓ. If $\delta\nu < 1 - h - m$ agents should all respond accepting when acceptance is low and rejecting otherwise so we can get both $\ell = 1$ or $\ell = 0$. In other words, there are two pure strategy Nash equilibria in this zone: full and zero investor participation, with the first of them Pareto preferable to the second $(1 - h - m > 0)$. If $\delta\nu > 1 - h - m$ agents should reject proposal for all ℓ.

For $\theta \in [\bar{\theta}, \hat{\theta}]$ sovereign pays and exits default $\forall \ell \geq 0$, so agents' participation would not risk any loss (in others' decisions). Agent's best response depends on payoff parameters. When $\delta\nu$ is such that $1 - h - m > \delta\nu$, accepting is the payoff dominant strategy, $\ell = 1$ yielding a full participation unique equilibrium. On the contrary, if $1 - h - m < \delta\nu$, rejecting the proposal is payoff dominant and $\ell = 0$, in a unique no participation equilibrium. Finally if $1 - h - m = \delta\nu$ agents are indifferent between accepting or rejecting and we obtain a unique mixed strategies equilibrium with $\ell \in [0, 1]$.

Lemma 2. *WLG, we can assume that in equilibrium $\delta\nu < 1 - h - m$ so that holdouts expected payment imposes to the government a limit when determining the haircut level which turns stage 2 into a two equilibria coordination game.*

Proof: If $\delta > 1 - h - m$ there is general rejection with $\ell = 0$, and unless $\theta > \bar{\theta}$, government would remain on default. A participating equilibrium would then be bounded below by off proposal payoff: $\delta\nu \leq 1 - h - m$. For agents to be indifferent, government policy should be to pay $1 - h = \delta\nu + m$, and in this case the game delivers an internal solution ($\ell \in (0, 1)$). That equilibrium, however has zero probability of occurrence, because even for a small ϵ fixing the proposal so that $1 - h + \epsilon - m = \delta\nu$ gets full acceptance. So government's convenience will drive to a full participating equilibrium by setting $1 - h - m > \delta\nu$ ■

A consequence of Lemma 2, is that the incentive to hold out the acceptance in this model results from agents avoiding to lose $-m$ in case the proposal fails, and not from an extra expected payment ($\delta\nu < 1 - h - m$ is obviously not a preferred payoff).

Summing up, in stage 2, each value of fundamental θ determines a bayesian subgame that solves into unique zero and full participation Nash equilibria outside $(\underline{\theta}, \bar{\theta}]$. However, values of θ inside that range produce multiplicity with both full participating and no participating simultaneous equilibria.

2.3.3 Stage 1

In this stage the government determines the optimal haircut h as the value that maximizes its expected utility $G^*(h)$ in θ. Remember that the exact value of the fundamental θ has not been revealed yet and, as a consequence, the government assumes it takes some value in $[\check{\theta}, \hat{\theta}]$.

In the previous stage, we could identify dominance regions in bondholder's strategic behavior. These however, yield multiple equilibria (all bondholders participate and sovereign exists default and no one participates and sovereign remains in default) which does not allow us to determine a unique result for the government's problem.

2.4 Incomplete information

In this section I introduce some noise in investors' information using a global games approach to refine multiplicity. The uncertainty about θ still remains, but once the nature makes a draw from the distribution of θ, the government observes it directly, while investors only receive a noisy signal of it. Then uncertainty affects both types of agents but incomplete information will only affect investors. The new features do not modify stage 3 so this section will focus on the dynamics of the model over stages two to one.

2.4.1 Stage 2

Now investors' information set is $\mathcal{I}^2_{in} = \{x_i\}$ where $x_i = \theta + \sigma\varepsilon_i$ corresponds to a noisy private realization of fundamental θ with ε_i independent standard normal perturbations and $\sigma > 0$ a scaling parameter. In this setting, a strategy $s_i(x_i)$, for creditor i is a decision rule that maps from the space of signals to that of actions: $\mathbb{R} \to \{0, 1\}$. Accordingly, an equilibrium is a profile of strategies that maximizes each creditor's expected payoff, conditional on the information available.

It is important to note that investor's payoffs still depend on the realized value of θ not on his private signal which literature calls common values model. Morris and Shin (2002), propose a general framework to solve such games consisting of a series of sufficient conditions for the payoffs gain function: $u(1, \ell, \theta) - u(0, \ell, \theta)$. For those payoff functions compliant, it is possible to obtain a strategy profile s that conforms a unique Bayesian Nash equilibrium of this game.

Proposition 1. *Let θ^* be defined as in (4).*

$$\theta^*(h) = \frac{1}{\xi}\left(1 - h + m\frac{\nu - (1-h)}{1 - h - \delta\nu}\right) \tag{4}$$

For any $\tau > 0$, there exists $\bar{\sigma} > 0$ such that for all $\sigma \leq \bar{\sigma}$, if strategy s_i survives iterated deletion of strictly dominated strategies, then $s_i(x_i) = 0$ for all $x_i \leq \theta^* - \tau$, and $s_i(x_i) = 1$ for all $x_i \geq \theta^* + \tau$.

Proof: Appendix C.

A take away from the demonstration of Proposition 1, is threshold $\theta^*(h) \in [\underline{\theta}, \bar{\theta}]$ in (4) which discriminates regions of strict dominance in the game: when noise converges to zero, agents with signals $x_i > \theta^*(h)$ accept proposal and those that observe signals below that level reject it.

Characterization of the threshold $\theta^*(h)$

Some algebra on (4) allows us to express the threshold $\theta^*(h)$ as a convex combination of government payments to each set of investors (accepting and rejecting proposal) in (5), with the ratio of net costs to benefits of accepting as weightings.

$$\theta^*(h) = \frac{1}{\xi}\left((1-h)(1 - \frac{m}{1 - h - \delta\nu}) + \nu\frac{m}{1 - h - \delta\nu}\right) \tag{5}$$

In this expression, we can see that low values of participation cost m move the threshold away from holdouts payment as they increase the probability of a high acceptance result.

Although $\theta^*(h)$ is determined by the complete set of parameters in the model, not all of them affect it through the same channel. For instance, m and δ contribute directly via agents' payoff function. ξ has an indirect effect on $\theta^*(h)$

by affecting government's threshold $\ell^*(\theta)$, and then modifying the total acceptance required to shift the state, and the probability of different payoffs with it. Finally, the haircut h exerts both a direct (through agents decisions) and an indirect (through $\ell^*(\theta)$) effect on the threshold. These relations are the key content of the corollary below.

Corollary 1. $\frac{\partial\theta(h)^*}{\partial\xi} \leq 0$, $\frac{\partial\theta(h)^*}{\partial\nu} \geq 0$, $\frac{\partial\theta(h)^*}{\partial\delta} \geq 0$, $\frac{\partial\theta(h)^*}{\partial} \geq 0$.

Proof: Appendix E.

Increases in holdout receipts, holdouts payments or participation costs discourage investors acceptance while higher after-restructuring boost (ξ) encourages it.

When participation costs m or holdout receipts δ rise, net participation benefit reduces (for higher costs or a better outside option) increasing the probability of rejection (as $\theta^*(h)$ expands). Note that high δ or m values compresses the haircut to the minimum, and even result in complete rejection in the limit, when the outside option is competitive or participation is too expensive.

Better after-restructuring conditions (higher ξ) make the program more attractive for the government whom then reduces the participation threshold $\ell^*(\theta)$ to augment its probability of success. This, in turn, reduces the chance of losing m for accepting a failed program and then encourages investor participation (by compressing θ^*). In the case of holdouts payments ν, they produce the opposite effect through the same channel, due to the fact that increases in ν force the government to tighten ℓ^*. As ν converges to $1-h$, $\theta^*(h)$ converges to $\underline{\theta} = \frac{1-h}{\xi}$, its lowest boundary, expanding the acceptance zone towards its maximum extension. ν at its lowest boundary, implies lower obligations for the government that reduces $\ell^*(\theta)$ in order to encourage participation to the maximum (as $\frac{\partial\ell^*(\theta)}{\partial\nu} \geq 0$ in $\theta \geq \frac{1-h}{\xi}$). On the contrary, $(1-h)$ converging towards ν pushes θ^* to $\bar{\theta}$ reducing the acceptance region to the minimum: in this case there would be little gain from the program and the government must require $\ell^*(\theta) = 1$, decreasing the probability of success.

θ as a function of the haircut

Corollary 2. *$\theta^*(h)$ is a U-shaped convex function that minimizes at:*

$$h_{in} = 1 - \delta\nu - \sqrt{m(1-\delta)\nu}$$

Proof: Appendix E.

So there is first an indirect effect through government's threshold ℓ^*. As h increases, government commits to pay a smaller share of defaulted debt and the relief allows it to reduce the minimal acceptance threshold for the proposal (3). For investors, that implies a contraction in the probability of losing m (when entering a failed proposal). As a consequence, the acceptance region increases by decreasing the lower limit $\theta^*(h)$. The direct effect, on the contrary, affects $\theta^*(h)$ through investors' payoff: a higher h, determines a reduction in marginal

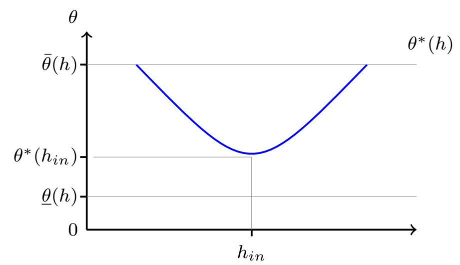

Figure 4: Threshold as a function of haircut.

utility of accepting the proposal thus discouraging investors that in consequence extend the non acceptance region (increasing $\theta^*(h)$).

Drawing upon convexity we can optimize $\theta^*(h)$ in h (appendix E). In this unique value, which we denominate h_{in}, the threshold reaches its minimum, implying that the accepting region expands most (and the probability of a successful proposal with it) for a given set of parameters.

$$1 - h_{in} - \delta\nu = \sqrt{m(1-\delta)\nu} \tag{6}$$

This result contrasts with the idea that every sufficiently low h coordinates investors in the full acceptance equilibrium: here, however, there is a trade off for h that originates in strategic behavior. Although bondholders' investment recovery rate increases with a lower haircut (so we would expect higher participation rate), at the same time government situation deteriorates and the threshold $\ell^*(\theta)$ is raised, demanding a higher market support for the program which in turn increases the probability of failure and the lose of m for those whom participated (determining a lower participation rate).

That h_{in} level, however, would not be attainable by the government. For instance, replacing (6) in (3) with $\ell^* = 1$ we obtain the lowest payment capacity required to exit default when the haircut is set at h_{in} level, which we will denote by θ_0.

$$\theta_0 = \frac{1}{\xi}\left(\delta + \sqrt{m(1-\delta)\nu}\right) > \frac{1}{\xi}\left(\delta + 2\sqrt{m(1-\delta)\nu} - 1\right) = \theta^*_{in} \tag{7}$$

As it shows in (7),[13] had the government set the haircut in h_{in}, it would enter default for every observed $\theta \in [\check{\theta}, \theta_0]$ which is larger than $[\check{\theta}, \theta_{in}]$. So at least at this level of haircut, the government constraint adds $\theta_{in} - \theta_0$ to the probability of keeping at default.

Another consequence of convexity in $\theta^*(h)$ is that for any h not equal to h_{in} there is a pair of values that results in the same $\theta^*(h)$, and as a consequence in the same acceptance level. The effect of this duality in government decision is the main content of Proposition 2.

[13] $\theta_{in} > \theta_0$ requires $\sqrt{m(1-\delta)\nu} > 1$ which would be false due to $1 \geq \nu > \delta \geq 0$ and m small.

Proposition 2. *The set of feasible haircut values* $\mathfrak{h}$ *for the government to solve its maximization problem is* $\mathfrak{h} = [h_{in}, 1 - \delta\nu - m)$.

Proof: Trivial as government's utility is an increasing function on h■

Observe finally, that the presence of m is critical to determine the optimal h. In this model, it is the existence of participation costs which forces both extreme values of h_{co} to differ from $1 - \delta\nu$. So increasing costs demand a lower haircut to keep investors entrance.

2.4.2 Stage 1

Having identified investors' optimal responses to θ, now the government computes the optimal haircut, as the value that maximizes its expected utility in (1) conditional to θ.

$$G^*(h) = \int_{\check{\theta}}^{\theta^*(h)} \theta f(\theta) d\theta + \int_{\theta^*(h)}^{\hat{\theta}} (\theta(1+\xi) - (1-h)) f(\theta) d\theta \tag{8}$$

We reduce the expression (8) by applying both simple and truncated expectation formulas.

$$G^*(h) = E[\theta] + \big(1 - F(\theta)\big)\big(\xi E[\theta|\theta > \theta^*] - (1-h)\big) \tag{9}$$

Differentiating (9) and solving for h, we obtain h_{co}, the haircut level that maximizes government's utility.

Proposition 3. h_{co} *solves:*

$$\frac{1 - F(\theta^*(h))}{f(\theta^*(h))} = \frac{\partial \theta^*(h)}{\partial h}\big(\xi\theta^*(h) - (1-h)\big) \tag{10}$$

Proof: Appendix F.

From the second derivative of (9) we extract the conditions for a unique solution, which relay explicitly on the distribution function of θ.

Proposition 4. *Necessary conditions to solve government's maximization problem into a unique value* h_{co}.

1. $\frac{\partial f(\theta^*(h))}{\partial \theta^*(h)} \geq 0$

2. $\frac{\frac{\partial f(\theta^*(h))}{\partial \theta^*(h)}}{f(\theta^*(h))} \leq \frac{\frac{\partial^2 \theta^*(h)}{\partial^2 h}}{\left(\frac{\partial \theta^*(h)}{\partial h}\right)^2}$

Proof: Appendix G.

The haircut as a function of model parameters

Corollary 3. *Under conditions stated in Corollary 4* h_{co} *is: decreasing in* ν, δ *and increasing in* ξ.

Proof: Appendix H.

The haircut reduces with holdouts expectation δ and participation costs m as the government tries to compensate bondholders whose outside option and costs increased to secure its participation. In the limit, zero costs allow the government to situate the haircut at the highest possible value $h_{co} = 1 - \delta\nu$.

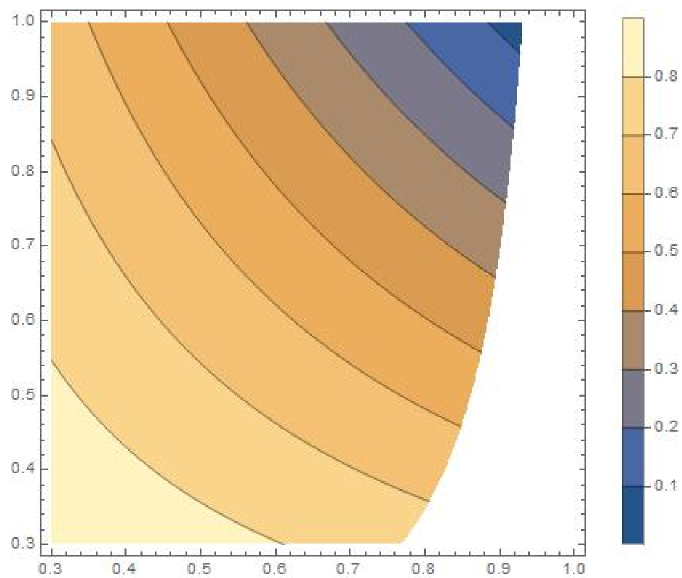

Figure 5: Simulated haircut on model parameters. Contour plot: h_{co} as a function of δ (abscissa) and ν (ordinate), with $m = 0.013$, $\xi = 0.04$, assuming an exponential distribution of θ with parameter $\lambda = 0.12$. White zone is restricted for $h > 1 - \delta\nu - m$.

Holdouts payments ν, also exerts a negative effect on the haircut through both government's and investor's strategic decisions. Unlike the previous case, increases in ν do not only affect investors utilities but the threshold $\theta^*(h)$: higher payments to holdouts reduce the probability of success which expands the rejection region. In addition, a higher ν erodes government net assets position which then boosts required participation ratio ℓ^*. In both cases, the government compensates discouraged investors by reducing h. This double negative effect shows in figure 5, where it is evident that δ has a slightly lower effect on h_{co}.

The haircut increases with post exit boom, via again both investors' and government's channels. In the first case, the exit boom ξ affects the threshold θ^* by increasing the acceptance zone once the government reduced $\ell^*(\theta)$ with the expected better results after the negotiation. Besides, the increase in post-negotiation boom encourages the government to reduce the haircut in order to secure the agreement and obtain a higher utility.

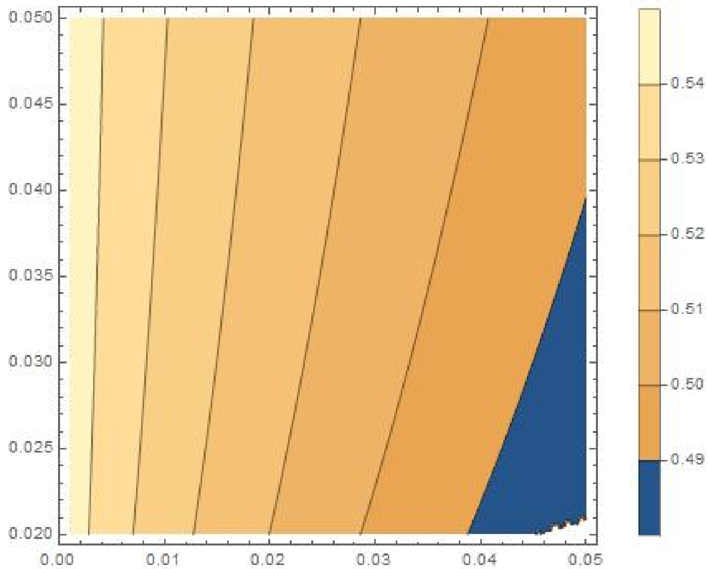

Figure 6: Simulated haircut on model parameters. Contour plot h_{co} as a function of m (abscissa) and ξ (ordinate), with $\delta = 0.66$, $\nu = 0.66$, assuming an exponential distribution of θ with parameter $\lambda = 0.16$. White zone is restricted for $h > 1 - \delta\nu - m$.

3 Model assessment

In this section I present two discussions regarding the equilibrium result. In the first part, there is a comparison between the coordination haircut and the haircut widely used in restructuring literature, obtained using Nash bargaining. I find that under certain conditions the Nash bargaining haircut overstates the coordination haircut and the difference increases with atomization of investors. In the second part, I propose a procedure to assess the cost for the government of changing from bank financing towards market financing, which we will call coordination costs. We find that these costs can reach a significant level in simulations, accounting for almost a fifth of the average difference between banks and bonds financing.

3.1 Coordination vs Nash bargaining haircut

Quantitative literature of sovereign debt with endogenous restructuring uses Nash bargaining to determine the equilibrium outcome of the restructuring process.[14] In particular, this tool assumes that the negotiation between investors

[14] Aguiar and Gopinath (2006) and Arellano (2008) assumed an exogenous recovery value after default (equal to zero or randomly set). Yue (2010) first proposes to endogenize the haircut through a Nash bargaining solution, and many authors use one-or-multiple-shot Nash

and the government is bilateral. This subsection aims to analyze whether the consideration of coordination strategies amongst investors can produce significant deviations in the equilibrium outcome. For this objective, I compute a regular Nash bargaining solution and another that incorporates coordination in this restructuring game. Finally, I compare both results in the bargaining power space.

Two additional concerns with the use of Nash bargaining in quantitative models derive from its critical reliance on the non-observed bargaining power attributed to each agent. First, when the model is used ex-ante to project an equilibrium haircut, the result depends on the bargaining power assumed. Second, when the model is used ex post, the bargaining power is calibrated to obtain an empirically observed haircut value. This is not trivial as changes in bargaining power might have significant effects on other variables. For instance, in Yue (2010) variations in the bargaining power of about 18% produce variations of 3% in the average debt to output ratio, 13% in the average recovery rates and significant changes in correlations. In contrast, the model in this paper delivers endogenously the bargaining power in terms of the parameters of the model (holdout premium, probability of repayment, expected recovery and expected fundamental value) which in turn can be traced to variables in the market. In addition, changes in market conditions are addressed through model parameters allowing for variations in bargaining power and thus in the restructuring outcome across time.

In order analyze the differences in equilibrium haircuts we start by expressing the generalized Nash product in terms of the payoffs of this model. Then, we will express both haircuts as a function of the bargaining power and compare the results in order to gain some intuition. (See Appendix J for details).

The generalized Nash product is presented in (11). α and $1-\alpha$ denote government's and investors' bargaining power respectively, and $\bar{\theta}$ denotes the expected value of fundamental θ. Government's payoff function in (1) provides the gains of restructuring and the outside option available, θ. In the case of bondholders, the outside option to participate yields a payoff of $\delta\nu$ as no agent alone can change the overall restructuring result (to get 0 in case proposal fails).

$$\omega(h) = \alpha\ell\big((1+\xi)\bar{\theta} - (1-h)\ell - \nu(1-\ell) - \bar{\theta}\big)^{\alpha}\big(1-h-m-\delta\nu\big)^{1-\alpha} \tag{11}$$

The Nash bargaining haircut in this setting h_{nb} is presented in (12) and the corresponding bargaining power α_{nb} in (13). This is the h value that maximizes the generalized Nash product of expected utilities. Note that homogeneity in investors' utility function implies that if proposal succeeds $\ell = 1$, which simplifies the expressions.

$$h_{nb} = \alpha(1-m-\delta\nu) - (1-\alpha)(\xi\bar{\theta} - 1) \tag{12}$$

$$\alpha_{nb} = \frac{\xi\bar{\theta} - (1-h)}{\xi\bar{\theta} - (m+\delta\nu)} \tag{13}$$

bargaining since then. See for instance Asonuma and Joo (2019), Bai and Zhang (2019).

For the Nash bargaining haircut with coordination strategies among investors, I use the result in Proposition 1. The main difference with the general model above is that now the Nash product $\omega(h)$ includes the truncated mean of the fundamental $\mathring{\theta}(\theta^*(h))$, instead of $\bar{\theta}$, adding a new dependence on h to consider during the maximization.

$$\omega(h) = \alpha\big((1+\xi)\,\mathring{\theta}(\theta^*(h)) - (1-h)\ell - \nu(1-\ell)\big)^{\alpha}\big(1-h-m-\delta\nu\big)^{1-\alpha} \tag{14}$$

I will call $h_{nb,co}$ the coordination haircut that solves equation (15), and $\alpha_{nb,co}$ its associated bargaining power in (16).

$$\alpha\big(1-h-m-\delta\nu\big)\left(\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h}+1\right)-(1-\alpha)\big(\xi\,\mathring{\theta}(\theta^*(h))-(1-h)\big)=0 \tag{15}$$

$$\alpha_{nb,co} = \frac{\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)}{\xi\,\mathring{\theta}(\theta^*(h)) + \big(1-h-m-\delta\nu\big)\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} - \delta\nu} \tag{16}$$

Lemma 3. *Under certain conditions, $h_{nb,co}$ and h_{nb} are both monotonous increasing functions of α.*

Proof: Appendix K.

Lemma 3 discusses monotonocity and slope properties of the haircuts obtained through the Nash bargaining process. Now we can use the previous results to map each haircut h_{nb} and $h_{nb,co}$ into the α space and compare them for each value of the bargaining power $\alpha \in [0,1]$.

In the next proposition, we state that there is a decreasing wedge between haircuts, when plotted agains α, and that the haircut that considers coordination strategies remains below the general one.

Proposition 5. *Under conditions in Lemma 3,*

1. *$h_{nb,co} \leq h_{nb}$ for all $\alpha \in [0,1]$.*
2. *$h_{nb} - h_{nb,co}$ is decreasing in α.*

Proof: Appendix J.

Using proposition 5 and the limit values of both measures of haircut we plotted h_{nb} and $h_{nb,co}$ against α in figure 7. Now let us look at the limit values. When government's bargaining power is complete, $\alpha = 1$, both measures yield a haircut of $1-\delta\nu-m$, the maximum feasible value for h_{co} according to Lemma 2. On the opposite extreme, when $\alpha = 0$ we can have one or many values for h depending on the distribution function of θ. Lemma 3 provides conditions, to identify monotonous h_{co} functions increasing over both $\alpha = 0$ and $\alpha = 1$ which intersect h_{nb} in $\alpha = 1$. In those cases, the Nash bargaining haircut situates above the coordination haircut.

This statement means that when using Nash bargaining to obtain an equilibrium outcome, we could be overestimating the haircut if the debt was issued in

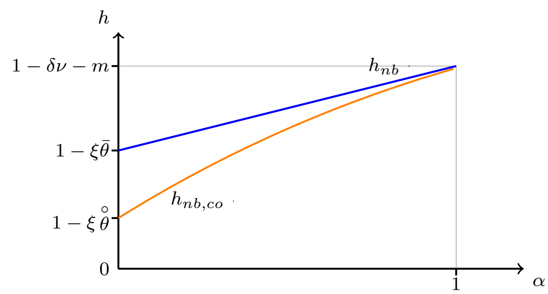

Figure 7: Nash and coordination haircut.

capital markets where investors play coordination strategies, and where there is no room for communication between them (to coordinate strategies at least among the main players). The size of this error increases with the bargaining power of investors.

Wrapping up, a more detailed micro founded model signals that there is a loss of bargaining power that derives from a negotiation against a group of bondholders playing a coordination game.

3.2 A measure of coordination costs

To measure the coordination costs we will compute the deviation of the equilibrium value from the maximum attainable haircut. Using Lemma 2, the latter coincides with the net outside option value for investors and the upper bound for h. The same result we get in a scenario in which the government keeps all the bargaining power $\alpha = 1$, but it confronts a unique investor instead of a continuous of unorganized agents. Thus in both cases we get $h_{nb,\alpha=1}$ in (17) and it then can be calculated coordination costs as the difference in both haircuts as in (18).

$$h_{nb,\alpha=1} = 1 - \delta\nu - m \tag{17}$$

$$coordination\, costs = h_{nb,\alpha=1} - h_{co} \tag{18}$$

Parameter values used in the simulations are presented in Table 1. The value of ν was set following the average recovery value in Cruces and Trebesch database plus the weighted average of holdout premium in Fridson and Gao (2002) and Altman and Eberhart (1994). δ value was selected in order to target market price $\delta\nu = 0.3$ as in the weighted average presented in Moody's investors service data report (2017) for 30-day post-default price or distressed exchange trading price. The value of ξ was set to 0.04 in what we understand would be a conservative value if we consider Das et al. (2012) average economy's recovery rates after restructuring. m is a non observable participating cost it was calibrated to be 1% in total investment $m = 0.010$.

Parameter		Value	Source
Holdout payment	ν	0.835	Holdout premium
Discount on holdout payment	δ	0.375	Moody's avge. prices
Recovery after restructuring	ξ	0.004	Das (2012)
Participation costs	m	0.010	

Table 1: Model parameters in simulations

Using the previous calibration, I simulate coordination costs as a function of junk market price variables δ and ν in Figure 8. There we can observe that costs situate in the 0-3.5% range, almost a 10% of average haircut reported in Cruces and Trebesch (2013). The coordination costs reduce with discount δ but increase with holdout payment ν. $h_{nb,\alpha=1}$ has a fixed reduction rate of δ with ν. The increase in coordination costs derive from a higher reduction rate of haircut h_{co}.

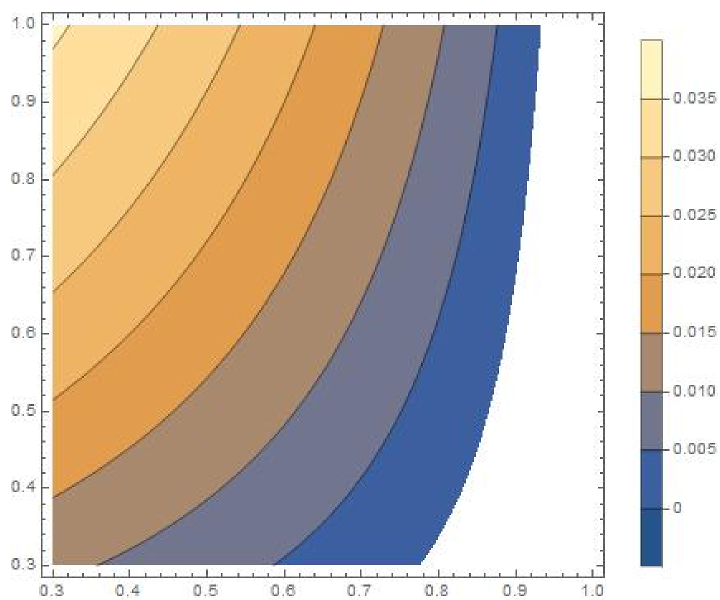

Figure 8: Coordination costs on model parameters. Contour plot: coordination costs as a function of δ (abscissa) and ν (ordinate), with $m = 0.01$, $\xi = 0.04$, assuming an exponential distribution of θ with parameter $\lambda = 0.16$. White zone is restricted for $h > 1 - \delta\nu - m$.

Figure 9, presents simulated coordination costs as a function of participation costs m and recovery after default ξ. Now the resulting costs range in 1% to 6% values near 20% of average observed haircut. Maximum values can be attained within reasonable parameters levels such as $\xi = 0.0275$ and m with in

[0.015, 0.03].

Although with some non linearities, for the higher values, costs reduce with ξ. This is an expected result because the haircut increases in ξ through (3) while this variable does not affect $h_{nb,\alpha=1}$. In the case of participation costs, they have an increasing effect on coordination costs. As they affect constantly $h_{nb,\alpha=1}$ (upwards) we can say that increases in m have an increasing reduction power on the haircut.

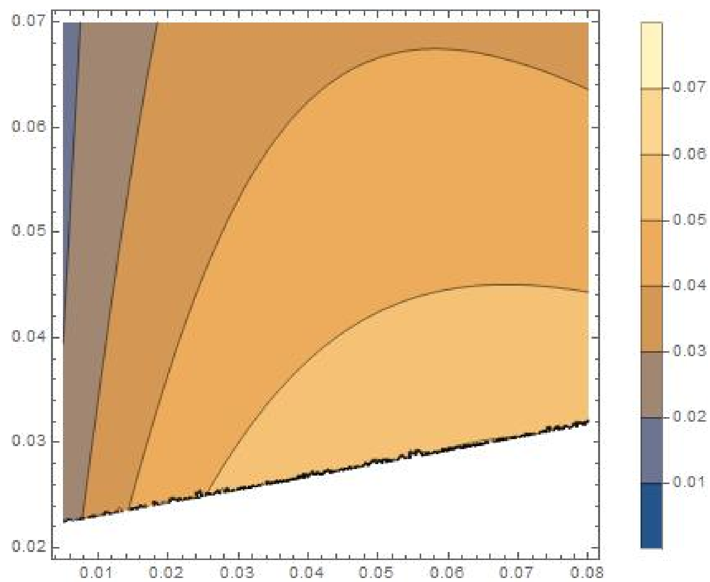

Figure 9: Coordination costs on model parameters. Contour plot: coordination costs as a function of m (abscissa) and ξ (ordinate), with $\delta = 0.375$, $\nu = 0.835$, assuming an exponential distribution of θ with parameter $\lambda = 0.16$. White zone is restricted for $h > 1 - \delta\nu - m$.

It is important to note that these results are in line with the fact that the disintermediation in sovereign debt financing determined a reduction in the haircut. In other terms, in this new market setting, the government has a cost derived from the fact that it does not bargain directly with its counter parties and instead it has to target spread investors and their higher order beliefs with the proposed haircut (all agents have to think that the rest will participate as it is an appropriate proposal). Costs around 7% count for almost 25% of the difference in banks vs bonds debt, which results a significant figure in terms of money losses for a government in distress.

4 Conclusions

After the strong financial disintermediation process that started in 1980, negotiating an exit from defaulted debt turned a complex process. It requires to align the decisions of many thousands of investors with limited information regarding the progress of the process in a relative short period of time. The final result relies on the aggregated behavior of that mass of spread and mostly unconnected agents which are simultaneously elaborating their strategies. Surprisingly, instead of benefiting the sovereign, investor coordination lent the terms and outcomes of the restructuring towards the bondholders, with shorter negotiations and lower haircuts.

Game theory applied to sovereign debt restructuring entails using backward induction into a multi stage game in which one of the stages contains a multiple equilibria coordination scheme. These particularly occur over non limit economic conditions, where internal non trivial solutions seem most plausible. I then apply global games to solve multiplicity, which requires among other things, allocating small participation costs and noise in investors' information sets. With multiplicity thus solved, the model endogenizes the haircut which is uniquely set in terms of the parameters, including the distribution of an economic fundamental. Working with homogeneous bondholders yields full participation avoiding contract coordination devices (special majorities, exit consents, etc).

I embed my model into a Nash bargaining structure, and find that under certain conditions, a pure Nash bargaining result obtains higher haircut values than the one with coordination for each possible level of bargaining power. This might be an important drawback to consider when using Nash bargaining to determine equilibrium outcomes in this new market setting (spread investors).

We can conclude that by introducing coordination strategies, we set a *de facto* constrain on government's bargaining power, as it is forced to reduce its proposal in order to coordinate bondholders towards the participation option (trying to align second order beliefs: here, it is what each investor think that other's think about accepting/rejecting the proposal). Using simulated results, coordination costs range in 0-7.0% of the proposed haircut, explaining a significant portion of the difference in banks vs bonds haircuts observed in data.

Finally, I would like to introduce a pair of comments about the model described above. First, it assumes homogeneous agents which results in a solution with full or none agreement. This simplifying assumption, works properly to explain massive retail behavior, and allows us to work with *plain-vanilla* instruments (with no contract amendments). On the other hand, it prevents the possibility of interior equilibria and, as a consequence, it cannot predict the participation rate. Second, I am modeling stakeholders decisions after default (which is the initial state), so the model does not include an assessment of sovereign's debt status at preliminar stages.

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Appendix

A Haircuts, some empirical facts

In this section I analyze some empirical facts from haircuts using the base of Cruces & Trebsech (2014 updated). Our variable of interest is the restructuring haircut (a measure of the total concession from debtors to creditors). The preferred measure is the one proposed in Sturzenegger & Zettelmeyer (2006 and 2008) in which the haircut summarizes the present value of investors' losses:

$$H^i_{SZ} = 1 - \frac{\text{Present value of New debt}(r^i_t)}{\text{Present value of old debt}(r^i_t)} \tag{19}$$

Where r^i_t is the yield prevailing at the time of the restructuring, which is considered a good proxy of debtor's default risk after the restructure.[15]

	Observations	Mean	SD	Mean	Max
SZ Haircut	187	.40	.28	-.098	.97
By type of creditor					
Bank debt restructuring	165	.37	.28	-.098	.97
Bond debt restructuring	22	.37	.22	.04	.76
By era					
1978–1989	99	.25	.19	-.098	.93
1990–1997	48	.51	.28	.03	.92
1998–2013	40	.52	.32	-.08	.97

Table 2: Haircut summary. Haircut measured as in Sturzenegger Zettelmeyer (2006) by type of creditor and era. Source: Cruces and Trebsesch (2013) updated with 2014 new data.

Table 2, presents a summary of results. As it can be noted, there were 187 sovereign restructurings during 1978-2013 with a mean haircut of 40%. A break up by type of creditor shows that there are only 22 cases of bond restructuring, although the average haircut does not seem to differ with its characteristic. Finally, the period when the process took place seems to be an important element. In fact, before 1990 haircuts where set around 25% while after that date they doubled up to 50% on average. These results suggest that the year in which the process took place will be a variable to control for in order to better understand the determinants of the haircut.

In Table 3, splitting the sample by era, I compare haircut means by type of creditor. With the whole sample, consistent with previous result, both groups bank and bond restructuring show no significant differences on mean averages.

[15] Cruces and Trebesch (2013).

Splitting the sample using the variable era, I find that most bond restructurings situate at recent years (19 out of 22 cases in 1998-2020). Within this period there is almost a 30% difference in the haircut negotiated in restructuring, which results a significant difference according to the test.

	Bank	Bond	Difference	p-value
All sample	0.37	0.38	0.00	0.96
N	165	22		
1998-2013	0.66	0.39	0.27	0.01
N	19	21		

Table 3: Mean haircut by type of creditor and era.

Table 4 presents regression analysis results for the logarithm of haircut when using bonds against bank loans controlling by era, and others (logarithms of GDP pc and GDP). As the reader can note, once controlled by year, creditors nature results significant at least at 5% level and with a persistent negative sign. A little algebra on results indicates that the ratio of haircuts in bonds to banks situates at 43% and 52% in models (1) and (2) respectively.

Haircut	(1)	(2)
Bond dummy	-0.84***	-0.64*
	(0.25)	(0.25)
Constant	Yes	Yes
Decade	Yes	Yes
Other controls	No	Yes
N	180	162
adj. R^2	0.16	0.29

Table 4: Model estimates for the logarithm of Haircut. Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Other controls: GDP and GDP per capita in logarithmic units.

B Two investors illustration

Consider a two investors game where each of them has full information about economic fundamental θ. The game is sketched in the tables below, with investor one's strategies presented in rows and investor two's in columns. Payoffs are ordered in rows inside each cell respectively.

Government proposes a concession program whose result depends on observed fundamental θ. This variable in time, determines the payoff matrix and finally game equilibria as detailed below.

If $\theta < \underline{\theta}$ then $\ell^*(\theta) > 1$ in (3), meaning that the program fails even at 100% acceptance. In this case, there are no benefits for accepting investors: acceptance has a cost of $-m$ while rejection costs 0. Payoffs matrix is presented in Table 5 that shows weakly dominance in the unique Nash equilibrium in pure strategies, $s = \{s_1, s_2\} = \{Reject, Reject\}$.

Investor 1 \ Investor 2	Accepts	Rejects
Accepts	$-m$ $-m$	$-m$ $\underline{0}$
Rejects	$\underline{0}$ $-m$	$\underline{0}$ $\underline{0}$

Table 5: Payoffs matrix $\theta \in [\check{\theta}, \underline{\theta})$

Draws of $\theta \in [\bar{\theta}, \hat{\theta}]$ produce $\ell^*(\theta) \leq 0$, unconditional restructuring. The game pays $1-h-m$ to the accepting investors and δ to the rejecting ones (assume here that $\delta < 1-h-m$). This game in Table 6, solves into a unique pure strategies Nash equilibrium, with weakly dominance in $s = \{s_1, s_2\} = \{Accept, Accept\}$.

Investor 1 \ Investor 2	Accepts	Rejects
Accepts	$\underline{1-h-m}$ $\underline{1-h-m}$	$\underline{1-h-m}$ $\delta\nu$
Rejects	$\delta\nu$ $\underline{1-h-m}$	$\delta\nu$ $\delta\nu$

Table 6: Payoffs matrix $\theta \in [\bar{\theta}, \hat{\theta}]$

Finally, Table 7 portrays $\theta \in [\underline{\theta}, \bar{\theta})$ and then $\ell(\theta)^* \in [0, 1]$. Suppose we can identify a θ_0 value under which $\ell^*(\theta) > 0.5$: government requires both investors accepting to exit default. In this case (left box in the table) we get two equilibria in strict dominant strategies with mirror behavior. Above that level of θ government exits default even if only one investor participates (right box in the table). Now acceptance is a weakly dominant strategy and we get an all participating equilibrium.

Investor 1 \ Investor 2	Accepts	Rejects
Accepts	$\underline{1-h-m}$ $\underline{1-h-m}$	$-m$ 0
Rejects	0 $-m$	$\underline{0}$ $\underline{0}$

θ low $\rightarrow \ell^* > 0.5$

Investor 1 \ Investor 2	Accepts	Rejects
Accepts	$\underline{1-h-m}$ $\underline{1-h-m}$	$\underline{1-h-m}$ $\delta\nu$
Rejects	$\delta\nu$ $\underline{1-h-m}$	$\delta\nu$ $\delta\nu$

θ high $\rightarrow \ell^* = 0.5$

Table 7: Payoffs matrix $\theta \in [\underline{\theta}, \theta]$

Remember a strategy is an action plan for every contingency. Then we have:

$$a_i = \{(Reject, Reject), (Accept, Reject), (Accept, Accept), (Accept, Accept)\}$$

Where elements represent contingent fundamental subsets $\{[\check{\theta}, \underline{\theta}), [\underline{\theta}, \theta_0), (\theta_0, \bar{\theta}], [\bar{\theta}, \hat{\theta}]\}$, and inside each parenthesis we portray optimal responses to others' accept and reject decisions respectively.

The multiplicity originates at the second region, where we can get indistinctly all accepting and all rejecting equilibria.

C Equilibrium uniqueness

From (2) we construct in (20) the action gain function $\pi(\ell, \theta) : [0, 1] \times \mathbb{R}^+ \to \mathbb{R}$ as $\pi(\ell, \theta) = u(1, \ell, \theta) - u(0, \ell, \theta)$.

$$\pi(\ell, \theta) \equiv \begin{cases} 1 - h - \delta\nu - m & \text{if} \quad \ell \geq \ell^* \\ \\ -m & \text{if} \quad \ell < \ell^* \end{cases} \tag{20}$$

According to (20), agents make a $1 - h - \delta\nu - m$ net profit for accepting the agreement when it proceeds, and a $-m$ net loss for accepting it when it fails. Now we demonstrate equation (20) compliance with each condition:

- **C1: Action monotonicity: incentive to choose action $a = 1$ is increasing in ℓ.**
  $\pi(l, \theta)$ is a step function in ℓ, discontinuous at $\ell = \ell^*(\theta)$. If $1-h-\delta\nu-m > -m$ then $\pi(\ell^{*+}, \theta) = 1 - h - \delta\nu - m > \pi(\ell^{*-}, \theta) = -m$ and the function is increasing in other players' actions, implying strategic complementarity in the game. If $1 - h - \delta\nu - m < -m$, $\pi(\ell, \theta)$ is decreasing in ℓ: rejecting government proposal is a strictly dominant strategy and the game turns into a strategic substitutes structure. Coordination equilibrium requires $1 - h - \delta\nu - m > 0 > -m$ to ensure action monotonicity■

- **C2: State monotonicity: the incentive to choose $a = 1$ is non decreasing in fundamental θ.**
  $\ell^*_\theta(\theta) < 0$ in (3), implies that increases in θ reduce the lower bound of $\pi(\ell^+, \theta)$, expanding the region where action $a = 1$ is a dominant strategy (given C1 compliance: $\pi(\ell^{*+}, \theta) > \pi(\ell^{*-}, \theta)$). This can be interpreted as: worse economic conditions weaken government requirement of minimal acceptance which increases probability of a successful process■

- **C3: Strict Laplacian state monotonicity: ensures there is a unique crossing for a player with Laplacian beliefs.** Intuitively, it derives from the fact that players in the game assume a uniform distribution to the proportion ℓ of other players choosing action $a = 1$.

$$\int_{\ell=0}^{\ell=1} \pi(\ell, \theta) d\ell = \int_{\ell=0}^{\ell=\ell^*(\theta)} -m \, d\ell + \int_{\ell=\ell^*(\theta)}^{\ell=1} 1 - h - \delta\nu - m \, d\ell = 0 \tag{21}$$

Equation (21) can be expressed as a linear function on θ with a unique solution in θ^* (appendix D).

$$\theta^* = \frac{1}{\xi}\left(1 - h + m\frac{\nu - (1-h)}{1 - h - \delta\nu}\right) \tag{22}$$

- **C4: Uniform limit dominance:** There exists a pair of values $\{\theta_0, \theta_1\} \in \mathbb{R}$, and $\varepsilon \in \mathbb{R}_{++}$, such that [1] $\pi(\ell,\theta) \leq -\varepsilon$ for all $\ell \in [0,1]$ and $\theta \leq \theta_0$; and [2] there exists θ_1 such that $\pi(\ell,\theta) > \varepsilon$ for all $\ell \in [0,1]$ and $\theta \geq \theta_1$.
From (3) we know that $\ell^*(\theta)$ is a linear function with value 1 at $\underline{\theta} = \frac{1-h}{\xi}$ (figure 1). Using $\ell^*_\theta(\theta) < 0$ in lemma 1 we can define $\theta_0 = \frac{1-h}{\xi} - \varepsilon$, such that $\ell^*(\underline{\theta}) = 1 + \varsigma$ (for ς very small) and as a consequence $\pi(\ell,\theta) = -m$ for all $\ell \in [0,1]$ and every $\theta \leq \theta_0$■

  The analogue demonstration can be done for [2], taking in this case $\theta_1 = \frac{\nu}{\xi} + \varepsilon$.

- **C5: Continuity:**$\int_{\ell=0}^{\ell=1} g(\ell)\pi(\ell,x)d\ell$ **is continuous with respect to signal** x **and density** g**.**
$\pi(\ell,x)$ presents only one point of discontinuity at $\ell = \ell^*$. Thus for a continuous density $g(\ell)$, discontinuity acquires zero mass, then the integral is weakly continuous■

- **C6: Finite expectations of signals:** $\int_{z=0}^{z=+\infty} zf(z)dz$ **is well defined for integration.**
With $z = \frac{x-\theta}{\sigma}$, $f(z)$ is defined as a continuous density function with $\int_{z=0}^{z=1} f(z) < +\infty$■

As payoff gain function complies with conditions C1-C6, Proposition 1 states that the game can be solved to deliver a unique equilibrium in terms of fundamental θ. (Morris and Shin (2002))

D Laplacian state monotonicity proof

There is a single cross on θ^* for action gain function π.

$$\int_{\ell=0}^{\ell=1} \pi(\theta,\ell)d\ell = \int_{\ell=0}^{\ell=\ell^*(h)} -m\, d\ell + \int_{\ell=\ell^*(h)}^{\ell=1} 1 - h - \delta\nu - m\, d\ell = 0$$

$$-m\ell^*(h) + (1 - h - \delta\nu - m)(1 - \ell^*(h)) = 0$$

Now we replace ℓ^* for the expression in (3).

$$(1 - h - \delta\nu - m) - (1 - h - \delta)\frac{\nu - \theta\xi}{\nu - (1-h)} = 0$$

$$(1-h-\delta\nu-m)(\nu-(1-h))-(1-h-\delta)(\nu-\theta\xi)=0$$

$$\theta=\frac{-(1-h-\delta\nu-m)(\nu-(1-h))+(1-h-\delta)\nu}{(1-h-\delta)\xi}$$

$$\theta^*=\frac{1}{\xi}\left(1-h+m\frac{\nu-(1-h)}{1-h-\delta\nu}\right)\blacksquare$$

E Analysis of threshold θ^* partial derivatives

To determine (23), (24) and(26) I use $\nu>1-h\geq\delta$, $m>0$ and $\xi\leq 1$:

$$\frac{\partial\theta^*(h)}{\partial m}=\frac{1}{\xi}\left(\frac{\nu-(1-h)}{1-h-\delta\nu}\right)\geq 0 \tag{23}$$

$$\frac{\partial\theta^*(h)}{\partial\xi}=-\frac{1}{\xi^2}\left(1-h+\frac{m(\nu-(1-h))}{1-h-\delta\nu}\right)\leq 0 \tag{24}$$

$$\frac{\partial\theta^*(h)}{\partial\delta}=\frac{1}{\xi}\left(\frac{m(\nu-(1-h))}{(1-h-\delta\nu)^2}\right)\geq 0 \tag{25}$$

$$\frac{\partial\theta^*(h)}{\partial\nu}=\frac{1}{\xi}\left(m\frac{(1-\delta)(1-h)}{(1-h-\delta\nu)^2}\right)\geq 0 \tag{26}$$

$$\frac{\partial\theta^*(h)}{\partial h}=\frac{1}{\xi}\left(-1+\frac{m(1-\delta)\nu}{(1-h-\delta\nu)^2}\right) \tag{27}$$

Convexity of $\theta^*(h)$ in h allows us to find optimal value h_{in} in (28) by equating its derivative to zero:

$$1-h_{in}-\delta\nu=\sqrt{m(1-\delta)\nu} \tag{28}$$

F Government optimal haircut

We want to find $h\in[0,1]$ that maximizes government's expected utility $G^*(h)=E_\theta[G(h)]$ with $\theta\sim p[\check{\theta},\hat{\theta}]$:

$$h_{co}=argmax_{h\in[0,1]}\{G^*(h)\} \tag{29}$$

Government expected utility is presented in (31). Note that for values of θ above θ^* all agents accept, so that $\ell=1$.

$$G^*(h)=\int_{\check{\theta}}^{\theta^*}\theta f(\theta)d\theta+\int_{\theta^*}^{\hat{\theta}}(\theta(1+\xi)-(1-h))f(\theta)d\theta \tag{30}$$

Some rearranging of terms yields:

$$G^*(h) = \int_{\check{\theta}}^{\hat{\theta}} \theta f(\theta) d\theta + \xi \int_{\theta^*}^{\hat{\theta}} \theta f(\theta) d\theta - (1-h) \int_{\theta^*}^{\hat{\theta}} \theta f(\theta) d\theta \tag{31}$$

$$G^*(h) = E[\theta] + \big(1 - F(\theta^*(h))\big)\big(\xi E[\theta|\theta > \theta^*] - (1-h)\big) \tag{32}$$

Taking first derivative on (32) in h:

$$\frac{\partial G^*(h)}{\partial h} = -f(\theta^*(h)) \frac{\partial \theta^*(h)}{\partial h} \big(\xi E[\theta|\theta > \theta^*(h)] - (1-h)\big) ...$$

$$+(1 - F(\theta^*(h)) \left(\underbrace{\frac{f(\theta^*(h))}{1 - F(\theta^*(h))} \frac{\partial \theta^*(h)}{\partial h} \xi \big(E[\theta|\theta > \theta^*(h)] - \theta^*(h)\big)}_{(a)} + 1 \right) = 0$$

Where I am using in (a) differentiation properties of truncated expectation.

$$\frac{\partial G^*(h)}{\partial h} = 1 - F(\theta^*(h)) + f(\theta^*(h)) \frac{\partial \theta^*(h)}{\partial h} (1 - h - \xi \theta^*(h)) \tag{33}$$

So the optimal haircut h_{co} is the h that solves the equation in (34).

$$1 - F(\theta^*(h)) = f(\theta^*(h)) \frac{\partial \theta^*(h)}{\partial h} \big(\xi \theta^*(h) - (1-h)\big) \tag{34}$$

G Concavity of government's problem

Taking the second derivative in (33) with respect to h:

$$\frac{\partial^2 G^*(h)}{\partial^2 h} = -f(\theta^*(h)) \frac{\partial \theta^*(h)}{\partial h} + \frac{\partial f(\theta^*(h))}{\partial \theta^*(h)} \left(\frac{\partial \theta^*(h)}{\partial h} \right)^2 \big(1 - h - \xi \theta^*(h)\big)$$

$$+f(\theta^*(h)) \frac{\partial^2 \theta^*(h)}{\partial^2 h} \big(1 - h - \xi \theta^*(h)\big) + f(\theta^*(h)) \frac{\partial \theta^*(h)}{\partial h} \left(-1 - \xi \frac{\partial \theta^*(h)}{\partial h} \right)$$

From proposition 2, we know that government will set $h \in [h_{in}, 1]$ where $\frac{\partial \theta^*(h)}{\partial h} \geq 0$. In this model, a successful proposal obtains $\ell = 1$ which, using indifference condition of equation (1), implies that $\xi\theta > 1 - h$. This inequality will hold for θ^* and all the θ values above it (where the economy exists default).

$$\frac{\partial^2 G^*(h)}{\partial^2 h} = \underbrace{-f(\theta^*(h)) \underbrace{\frac{\partial \theta^*(h)}{\partial h}}_{\geq 0} \left(\underbrace{2 + \xi \frac{\partial \theta^*(h)}{\partial h}}_{>0} \right)}_{\leq 0}$$

$$+\left(\frac{\partial f(\theta^*(h))}{\partial\theta^*(h)}\underbrace{\left(\frac{\partial\theta^*(h)}{\partial h}\right)^2}_{\geq 0}+\underbrace{f(\theta^*(h))\frac{\partial^2\theta^*(h)}{\partial^2 h}}_{\geq 0}\right)\underbrace{\left(1-h-\xi\theta^*(h)\right)}_{\leq 0}$$

From the last expression, we can extract conditions for a global maximum that solves government's problem with uniqueness. Note that $\frac{\partial f(\theta^*(h))}{\partial h}$ plays a critical role.

H Analysis of h_{co} partial derivatives

To analyze partial effects of model parameters on optimal haircut over $[\check{\theta},\hat{\theta}]$, I will use implicit function theorem.

For this section, let us denote a vector $\mathbf{x}$ of model parameters such that $\mathbf{x}=[\xi,\nu,\delta,m]$ using x_i to refer to one of its elements. Thus, equation (10) can be written as follows:

$$1-F(\theta^*(h,\mathbf{x}))=f(\theta^*(h,\mathbf{x}))\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\left(\xi\theta^*(h,\mathbf{x})-(1-h)\right) \tag{35}$$

Total differentiating (35) with respect to x_i yields:

$$-f(\theta^*(h,\mathbf{x}))\left(\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\frac{dh}{dx_i}+\frac{\partial\theta^*(h,\mathbf{x})}{\partial x_i}\right)=$$

$$\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial\theta^*(h,\mathbf{x})}\left(\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\frac{dh}{dx_i}+\frac{\partial\theta^*(h,\mathbf{x})}{\partial x_i}\right)\left(\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\right)\left(\xi\theta^*(h,\mathbf{x})-(1-h)\right)$$
$$+f(\theta^*(h,\mathbf{x}))\left(\frac{\partial^2\theta^*(h,\mathbf{x})}{\partial^2 h}\frac{dh}{dx_i}+\frac{\partial\theta^*(h,\mathbf{x})}{\partial h\partial x_i}\right)\left(\xi\theta^*(h,\mathbf{x})-(1-h)\right)$$
$$+f(\theta^*(h,\mathbf{x}))\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}(\xi\left(\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\frac{dh}{dx_i}+\frac{\partial\theta(h,\mathbf{x})}{\partial x_i}\right)+\frac{dh}{dx_i})$$

Now let us solve for $\frac{dh}{dx_i}$ and observe the signs for the general case in (36):

$$\frac{dh}{dx_i}=\frac{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial\theta^*(h,\mathbf{x})}\frac{\partial\theta^*(h,\mathbf{x})}{\partial x_i}\underbrace{\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}}_{\geq 0}\gamma+\underbrace{f(\theta^*(h,\mathbf{x}))}_{\geq 0}\left(\frac{\partial^2\theta^*(h,\mathbf{x})}{\partial h\partial x_i}\underbrace{\gamma}_{\geq 0}+\underbrace{\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}}_{\geq 0}(1+\xi\frac{\partial\theta^*(h,\mathbf{x})}{\partial x_i})\right)}{-\underbrace{f(\theta^*(h,\mathbf{x}))}_{\geq 0}\left(\underbrace{\frac{\partial^2\theta^*(h,\mathbf{x})}{\partial^2 h}}_{>0}\gamma+\underbrace{\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\left(\xi\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}+2\right)}_{\geq 0}\right)-\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial\theta^*(h,\mathbf{x})}\underbrace{\left(\frac{\partial\theta^*(h)}{\partial h}\right)^2\gamma}_{\geq 0}} \tag{36}$$

Where I am using Proposition 2 in $\frac{\partial\theta^*(h,\mathbf{x})}{\partial h}\geq 0$. I am also using the fact that in equilibrium $\ell=1$ and that the proposal must comply indifference condition on government's utility in (1) so that $\xi\theta^*(h, \mathbf{x} - (1-h))$, which I will refer as γ in the rest of this section, is higher or equal than zero . In addition, I use the property of positive probability density functions.

Following, I discuss the response of coordination haircut according to the slope of the density function of fundamental θ.

H.1 Density derivative $\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x})}$ equals zero

In this case, the results will depend on the signs of $\frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i}$ and the crossed derivative of $\theta^*(h, \mathbf{x})$ on parameters, all of them affecting the numerator of the expression in (37), while the denominator is negative. I additionally use the fact that $\theta^*(h, \mathbf{x})$ is a U-shaped parabola in the second derivative respect to h.

$$\frac{dh}{dx_i} = \frac{\left(\frac{\partial^2\theta^*(h,\mathbf{x})}{\partial h \partial x_i}\underbrace{\gamma}_{\geq 0} + \frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i}\left(\underbrace{\frac{\partial \theta^*(h,\mathbf{x})}{\partial h}\xi + 1}_{>0}\right)\right)}{\underbrace{\underbrace{\frac{\partial^2\theta^*(h,\mathbf{x})}{\partial^2 h}\gamma}_{\geq 0} + \underbrace{\frac{\partial \theta^*(h,\mathbf{x})}{\partial h}\left(\xi\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} + 2\right)}_{\geq 0}}_{<0}} \tag{37}$$

Lemma 4. *Regarding the cross-second derivatives of $\theta^*(h, \boldsymbol{x})$:*

$$\frac{\partial^2\theta^*(h,\boldsymbol{x})}{\partial h \partial m} \geq 0, \; \frac{\partial^2\theta^*(h,\boldsymbol{x})}{\partial h \partial \nu} \geq 0, \; \frac{\partial^2\theta^*(h,\boldsymbol{x})}{\partial h \partial \delta} \geq 0, \; \frac{\partial^2\theta^*(h,\boldsymbol{x})}{\partial h \partial \xi} \leq 0$$

Proof: Appendix I

1. Sub-case $\frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i} > 0$ corresponds to m, ν, δ according to Corollary (1). Because of Lemma 4 we know that crossed derivatives are greater or equal than zero and as a consequence $\frac{dh}{dm} \leq 0$, $\frac{dh}{d\nu} \leq 0$, $\frac{dh}{d\delta} \leq 0$.

2. Sub-case $\frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i} \leq 0$, which corresponds to $x_i = \xi$ according to Corollary 1. Using Proposition 2, we know that numerator in (37) is negative and so $\frac{dh}{d\xi} \geq 0$.

H.2 Density derivative $\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x})}$ is positive

$$\frac{dh}{dx_i} = \frac{\underbrace{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x})}}_{>0} \frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i} \underbrace{\frac{\partial \theta^*(h,\mathbf{x})}{\partial h}}_{\geq 0} \gamma + f(\theta^*(h,\mathbf{x})) \left(\frac{\partial^2 \theta^*(h,\mathbf{x})}{\partial h \partial x_i} \underbrace{\gamma}_{\geq 0} + \frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i} \left(\underbrace{\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \xi + 1}_{>0} \right) \right)}{\underbrace{- \underbrace{f(\theta^*(h,\mathbf{x}))}_{\geq 0} \left(\underbrace{\frac{\partial^2 \theta^*(h,\mathbf{x})}{\partial^2 h}}_{\geq 0} \gamma + \underbrace{\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \left(\xi \frac{\partial \theta^*(h,\mathbf{x})}{\partial h} + 1 \right)}_{>0} \right) - \underbrace{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x})}}_{>0} \underbrace{\left(\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \right)^2 \gamma}_{\geq 0}}_{<0}} \tag{38}$$

Using lemma 4, as in the previous case we get $\frac{dh}{d\nu} \leq 0, \frac{dh}{dm} \leq 0$, $\frac{dh}{d\delta} \leq 0$ and $\frac{dh}{d\xi} \geq 0$.

H.3 Density derivative $\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x})}$ is negative

In this case, Proposition 5 for a unique solution to government's maximization problem (conditions for a concave government's expected utility) secures that denominator in (39) will always be negative, so let us now look the numerator to conclude.

$$\frac{dh}{dx_i} = \frac{\underbrace{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x}))}}_{<0} \frac{\partial \theta(h,\mathbf{x})}{\partial x_i} \underbrace{\frac{\partial \theta^*(h,\mathbf{x}))}{\partial h}}_{\geq 0} \gamma + f(\theta^*(h,\mathbf{x})) \left(\frac{\partial^2 \theta^*(h)}{\partial h \partial x} \underbrace{\gamma}_{\geq 0} + \frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i} \left(\underbrace{\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \xi + 1}_{>0} \right) \right)}{\underbrace{- \underbrace{f(\theta^*(h,\mathbf{x}))}_{\geq 0} \left(\underbrace{\frac{\partial^2 \theta^*(h,\mathbf{x})}{\partial^2 h}}_{\geq 0} \gamma + \underbrace{\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \left(\xi \frac{\partial \theta^*(h,\mathbf{x})}{\partial h} + 1 \right)}_{\geq 0} \right) - \underbrace{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x})}}_{<0} \underbrace{\left(\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \right)^2 \gamma}_{\geq 0}}_{<0}} \tag{39}$$

In this case, we get again $\frac{dh}{d\nu} \leq 0, \frac{dh}{dm} \leq 0$, $\frac{dh}{d\delta} \leq 0$ and $\frac{dh}{d\xi} \geq 0$ only in the case that condition (40) holds.

$$\frac{\frac{\partial f(\theta^*(h,\mathbf{x}))}{\partial \theta^*(h,\mathbf{x})}}{f(\theta^*(h,\mathbf{x}))} \leq \frac{\frac{\partial^2 \theta^*(h,\mathbf{x})}{\partial h \partial x_i} \gamma + \frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i} \left(\frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \xi + 1 \right)}{\frac{\partial \theta^*(h,\mathbf{x})}{\partial x_i} \frac{\partial \theta^*(h,\mathbf{x})}{\partial h} \gamma} \tag{40}$$

Meaning that in those cases in which the density function of θ presents a negative slope, for the partial derivatives of haircut respect to parameters to hold, it is needed a regular behavior in peakedness[16] of the distribution function.

[16]This characteristic is not equivalent to the kurtosis as this last one regards also the tails of the distribution while peakedness don't.

In other words, not a small set of values in the distribution with too high frequencies.

I Cross derivatives

In this appendix I demonstrate Lemma 4 of cross derivatives of $\theta^*(h, \mathbf{x})$. Using $\mathbf{x}$ as a vector of parameters ad x_i denoting one of its elements.

$$\frac{\partial^2 \theta^*(h, \mathbf{x})}{\partial h \partial m} = \frac{1}{\xi} \frac{(1-\delta)\nu}{(1-h-\delta\nu)^2} \geq 0$$

The case of $\frac{\partial^2 \theta^*(h,\mathbf{x})}{\partial h \partial m} = 0$ is for $\delta = 1$.

$$\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \nu} = \frac{1}{\xi} \frac{m(1-\delta)(1-h-\delta\nu)^2 - 2(1-h-\delta\nu)(-\delta)m(1-\delta)\nu}{(1-h-\delta\nu)^4}$$

$$\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \nu} = \frac{m(1-\delta)(1-h-\delta\nu)}{\xi} \left(\frac{1-h+\delta\nu}{(1-h-\delta\nu)^4} \right) \geq 0$$

So the cases of $\frac{\partial^2 \theta(h,\mathbf{x})}{\partial h \partial \nu} = 0$ are for $\delta = 1$ or $m = 0$.

$$\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \delta} = \frac{1}{\xi} \frac{-m\nu(1-h-\delta\nu)^2 - 2(1-h-\delta\nu)(-\nu)m(1-\delta)\nu}{(1-h-\delta\nu)^4}$$

$$\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \delta} = \frac{m\nu(1-h-\delta\nu)}{\xi} \left(\frac{\nu - (1-h) + (1-\delta)\nu}{(1-h-\delta\nu)^4} \right) \geq 0$$

Where the case of $\frac{\partial^2 \theta(h,\mathbf{x})}{\partial h \partial \delta} = 0$ is for $m = 0$.

$$\frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \xi} = -\frac{1}{\xi^2} \left(-1 + \frac{m(1-\delta)\nu}{(1-h-\delta\nu)^2} \right) \frac{\partial^2 \theta(h, \mathbf{x})}{\partial h \partial \xi} = -\frac{1}{\xi^2} \left(\frac{-(1-h-\delta\nu)^2 + m(1-\delta)\nu}{(1-h-\delta\nu)^2} \right) \leq 0$$

J Nash bargaining haircut

In this section we introduce our setting into a Nash generalized bargaining process, to now obtain the haircut as the allocation that maximizes Nash product in (41). $\omega(h)$ represents net output (restructuring success against failure) to distribute between agents: government and bondholders. Each of them has some bargaining power α and $1-\alpha$ respectively which will determine its share of the total product.

$$\omega(h) = (\bar{\theta}(1+\xi) - [(1-h)\ell + \nu(1-\ell)] - \theta)^{\alpha}(1-h-m-\delta\nu)^{1-\alpha} \tag{41}$$

It is important to note that if one investor i does not participate, it would not change the result of the agreement. So its individual outside option is to receive $\nu\delta$ and not zero (as the case in which the proposal fails).

J.1 General case

I obtain the haircut as the allocation that maximizes the generalized Nash product $\omega(h)$ of expected utilities in θ.

$$h = argmax\{\omega(h)\}$$

I am using $\bar{\theta}$ to denote the mean of the random variable θ. Now let us take the derivative of (41) in h and then solve for h.

$$\alpha\ell(\xi\bar{\theta} - (1-h)\ell - \nu(1-\ell))^{\alpha-1}(1-h-m-\delta\nu)^{1-\alpha} = \\ (1-\alpha)(\xi\bar{\theta} - \nu - \ell(1-h-\nu))^{\alpha}(1-h-m-\delta\nu)^{-\alpha} \quad (42)$$

Using that in a succesful proposal participation rate is $\ell = 1$, I get:

$$h_{nb} = \alpha(1-m-\delta\nu) - (1-\alpha)(\xi\bar{\theta} - 1) \quad (43)$$

We can get the α value for the equilibrium h as well.

$$\alpha = \frac{\xi\bar{\theta} - \nu - \ell(1-h-\nu)}{(\ell(-m-\delta\nu) + \xi\bar{\theta} - \nu - \ell(-\nu))} \quad (44)$$

With $\ell = 1$:

$$\alpha_{nb} = \frac{\xi\bar{\theta} - (1-h)}{\xi\bar{\theta} - (m+\delta\nu)} \quad (45)$$

J.2 Coordination case

We can think of a Nash Bargaining case where we introduce the coordination feature. In that case, we would use the truncated mean of the payment capacity θ instead of the plain mean, because for the agreement to succeed we need agents participating and that only occurs for those values of θ above or equal $\theta^*(h)$. This feature adds a new dependence link of $\omega(h)$ to h through $\theta^*(h)$. I will denote $\mathring{\theta}(\theta^*(h))$ the truncated mean and will take again the first derivative on (41) considering the new features.

$$\alpha\big(\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)\ell - \nu(1-\ell)\big)^{\alpha-1}\big(1-h-m-\delta\nu\big)^{1-\alpha}\left(\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} + \ell\right) \\ -(1-\alpha)\big(\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)\ell - \nu(1-\ell)\big)^{\alpha}\big(1-h-m-\delta\nu\big)^{-\alpha} = 0$$

After some arrangements, using equilibrium $\ell = 1$, we get:

$$\alpha\big(1-h-m-\delta\nu\big)\left(\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} + 1\right) - (1-\alpha)\big(\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)\big) = 0 \quad (46)$$

From were it is possible to obtain a new $\alpha = \alpha_{nb,co}$ that incorporates the coordination aspect of this problem:

$$\alpha_{nb,co} = \frac{\xi\,\mathring{\theta}(\theta^*(h)) - (1-h)}{\xi\,\mathring{\theta}(\theta^*(h)) + \big(1-h-m-\delta\nu\big)\xi\frac{\partial\,\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} - \delta\nu}$$

K Slopes of the haircuts under Nash bargaining model

In the general case, we can directly take the derivative of h_{nb} in α:

$$\frac{\partial h_{nb}}{\partial \alpha} = -m - \delta\nu + \xi\bar{\theta} > 0$$

In the coordination case, we have to use the implicit derivative of (46). As in appendix H, I denote $\gamma = 1 - h - \delta\nu - m$ to simplify notation. I obtain $\frac{dh}{d\alpha}$ below.

$$\frac{dh}{d\alpha} = -\frac{\gamma\left(\xi\frac{\partial \mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h}\right) + \xi\,\mathring{\theta}(\theta^*(h)) - (1-h)}{\gamma\xi\alpha\left(\frac{\partial^2\mathring{\theta}(h)}{\partial\theta^*(h)}\left(\frac{\partial\theta^*(h)}{\partial h}\right)^2 + \frac{\partial\mathring{\theta}(h)}{\partial\theta^*(h)}\frac{\partial^2\theta^*(h)}{\partial h}\right) - \left(\xi\frac{\partial\mathring{\theta}(\theta^*(h))}{\partial\theta^*(h)}\frac{\partial\theta^*(h)}{\partial h} + 1\right)}$$

The numerator in $\frac{dh}{d\alpha}$ is always non-negative. We have $\gamma > 0$ for Lemma 2, then the parenthesis is non negative for the definition of truncated mean and Corollary 2, and finally we have that $\xi\,\mathring{\theta}(\theta^*(h)) > 1 - h$. To see that, I expand the expressions using the derivative of the truncated mean of a random variable and denoting $H(\theta) = \frac{f(\theta)}{1-F(\theta)}$ to denote the hazard rate.

$$\gamma H(\theta^*(h))\big(\xi\,\mathring{\theta}(\theta^*(h)) - \xi\theta^*(h)\big)\frac{\partial\theta^*(h)}{\partial h} + \xi\,\mathring{\theta}(\theta^*(h)) - (1-h)$$

Note that $h = max\{0, 1 - \xi\,\mathring{\theta}(\theta^*(h))\}$. If $\xi\,\mathring{\theta}(\theta^*(h)) = 1 - h$, using the expression for $\theta^*(h)$ then we get $\mathring{\theta}(\theta^*(h) - \theta^*(h) < 0$ in (47), which contradicts the definition of truncated mean, thus implying that $\xi\,\mathring{\theta}(\theta^*(h)) > 1 - h$.

$$\xi\big(\mathring{\theta}(\theta^*(h)) - \theta^*(h)\big) = \big(\xi\,\mathring{\theta}(\theta^*(h)) - \xi\theta^*(h)\big) = -m\frac{\nu - (1-h)}{1 - h - \delta\nu} < 0 \qquad (47)$$

So up to now, we know that the coordination haircut as a function of α is increasing when $\alpha = 0$ and that it can have both an increasing or decreasing slope when $\alpha = 1$, where $h_{co} = 1 - \delta\nu - m$. In those cases in which (48) holds, $h_{\alpha,co}$ is a convex increasing function in α.

$$\gamma\xi\alpha\frac{\partial^2\mathring{\theta}(h)}{\partial\theta^*(h)}\left(\frac{\partial\theta^*(h)}{\partial h}\right)^2 < 1 + \xi\frac{\partial\mathring{\theta}(\theta^*(h))}{\partial\theta^*(h)}\left(\frac{\partial\theta^*(h)}{\partial h} - \gamma\alpha\frac{\partial^2\theta^*(h)}{\partial h}\right) \qquad (48)$$

L Comparison of cases inside Nash bargaining model

Using (43) and (46) we obtain $h_{nb}(\alpha = 0) = max\{0, 1 - \xi\bar{\theta}\}$ and $h_{nb,co}(\alpha = 0) = max\{0, 1 - \xi\,\mathring{\theta}\}$. For all those cases in which $1 - \xi\bar{\theta} > 0$, as $\bar{\theta} \leq \mathring{\theta}$ then $h_{nb}(\alpha = 0) > h_{nb,co}(\alpha = 0)$.

As I did before, we can get $h_{nb}(\alpha = 1) = 1 - m - \delta\nu$. For the coordination case, note first that using Corollary 2 and truncated mean properties we can get in (15) :

$$\xi \frac{\partial \mathring{\theta}(\theta^*(h))}{\partial \theta^*(h)} \frac{\partial \theta^*(h)}{\partial h} + 1 = \left(\frac{f(\theta^*(h))}{1 - F(\theta^*(h))} \left(\mathring{\theta}(\theta^*(h)) - \theta^*(h) \right) \right) \frac{\partial \theta^*(h)}{\partial h} + 1 > 0 \forall h$$

So now we can state that when $\alpha = 1$ there is a unique h value in which (15) holds: $h_{nb,co}(\alpha = 1) = 1 - m - \delta\nu$.

Finally, under Lemma 3 condition, intersection between both haircut functions is unique in $\alpha \in [0, 1]$ and in particular it occurs when $\alpha = 1$. As a consequence $h_{nb,co} \leq h_{nb}$ $\forall \alpha \in [0, 1]$. Convexity in $h_{nb,co}$ and the values in $\alpha = 0$ and $\alpha = 1$ allow us to confirm that the difference is decreasing in α.