



The Bright Side of the Doom Loop: Banks Exposure and Default Incentives

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The Bright Side of the doom loop: Banks' Exposure and Default Incentives

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Abstract

The feedback loop between sovereign and financial sector solvency has been identified as a key driver of the European debt crisis and has motivated an array of policy proposals. We revisit this “doom-loop” focusing on the government’s incentives to default. To this end we present a simple 3-period model with strategic sovereign default where debt is held by domestic banks and foreign investors. The government maximizes domestic welfare, and thus the temptation to default increases in foreign debt. Importantly, the costs of default arise endogenously from the damage default causes to domestic banks’ balance sheets. Domestically held debt thus serves a commitment device for the government. We show that two policy prescriptions that have emerged in this literature – lower exposure of banks to domestic sovereign debt or a commitment not to bailout banks – can backfire, as default incentives depend not just on the quantity of debt but also on who holds the debt. By contrast, allowing banks to buy additional sovereign debt in times of sovereign distress can rule out the doom loop.

Keywords: Sovereign Default; Bailout; doom loop; Self-fulfilling Crises

JEL Classification E44, E6, F34.

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1 Introduction

The “doom loop” or “sovereign-bank nexus” has been identified as a key driver of the European debt crisis and is recalled again with awe now that the response to the public health crisis causes sovereign debt to skyrocket.¹ According to this view, problems of sovereign debt sustainability and of financial stability reinforce each other due to mutual exposures between the public and the financial sectors. If public debt loses value due to deteriorating creditworthiness of the public sector, this hurts financial sector balance sheets, since the financial sector is a major holder of public debt. Weakened balance sheets in turn force the government to bailout the financial system – banks for short. This implies an expense for the government and hence a further deterioration of its fiscal capacity.² This vicious circle can amplify fundamental shocks (Acharay et al., 2014, Farhi and Tirole, 2016) or even give rise to crises that are purely generated by self-fulfilling pessimistic expectations (Brunnermeier et al., 2016, Brunnermeier et al., 2017, Cooper and Nikolov, 2018). It can hence explain why the sovereign crises can develop so suddenly and easily spiral out of control.

Based on this analysis a number of proposals to prevent such crises have been made. Cooper and Nikolov (2018) propose that a commitment not to bailout banks would reduce the exposure of the government to the financial sector and hence rule out self-fulfilling crises. Brunnermeier et al. (2016, 2017) suggest to reduce the exposure of domestic banks to the government by reducing the amount of public debt relative to their equity or by pooling and tranching debt of several sovereigns to the same end.

However, while the existing Doom Loop theory highlights how the financial sector’s exposure to the sovereign affects the government’s incentives to bail out banks and thus *indirectly* public debt sustainability, it ignores how the same exposure interacts *directly* with its incentives to repay. Indeed, *ceteris paribus*, a larger exposure of the domestic financial sector to the sovereign not only makes the government more prone to engage in larger bailouts, thus creating the potential for the doom loop (*indirect effect*). A larger exposure of the domestic financial sector also has a *direct effect* through two channels: First, if domestic agents hold more debt, then foreign investors hold less debt. Second, if domestic banks hold large amounts of sovereign debt, then default jeopardizes the functioning of the domestic banking system. Through these two channels, a larger exposure of domestic banks makes default less attractive. We refer to them as *temptation channel* and *commitment channel*. This effect thus counteracts the Doom Loop. In this paper we revisit the Doom Loop in a model which takes these interactions into account. Our theory puts the desirability of the above-mentioned policies to break the doom loop into question. Such policies may fail or come at a cost that makes them undesirable, whereas policies that appear counterproductive, such as allowing banks to load up on domestic debt, may be beneficial. So while the health crisis may indeed justify concerns about the doom loop, the right policy response to the loop is far from obvious. Contrary to the intuition underlying the

¹E.g. Brunnermeier (2015), Benassy-Quere et al. (2018)

²As Brunnermeier et al. (2016) argue, weaker balance sheets also affect the public sector indirectly by causing a credit crunch, which leads to lower output and hence a reduction in the tax base.

doom loop the fact that the new debt is mainly held by the domestic financial system may actually be desirable.

We develop these arguments in a simple 3 period model of sovereign debt and banks with multiple equilibria similar to the above cited papers. In period 0 the government issues debt, which is bought by foreign investors and domestic banks. Banks furthermore make loans. They finance their investments by deposits and equity. In period 1 a sunspot shock may hit the economy. If it does, the bond price may drop to a lower value, causing banks to fail. Bank failure entails a cost, because a fraction of the bank's loans get destroyed in that event. Thus the government has an incentive to intervene and bail them out. To do so it needs to issue more debt, which, in our baseline model, is bought by foreign investors. In period 2 all debt matures. The government defaults strategically but in a nondiscriminatory way. Sovereign default is costly in two ways: First, default causes output to drop. Second, if default bankrupts banks, it entails an additional cost in terms of forgone investment. The model features two equilibria: one where the materialization of a bad sunspot triggers the doom loop – a drop in the bond price causes a bank bailout, which increases debt and hence makes default more likely, thus validating the initial drop in bond prices; and one where this doesn't happen.

We then analyze several policy options. Here we proceed in three steps. In the first step we analyze policies that do not succeed at disabling the doom loop and contrast them to a policy that does. First, we find that it is not desirable to reduce the exposure of banks to the government at the margin. Such a policy leaves the doom loop intact but implies that foreign lenders hold more debt. The latter reduces the commitment to repay and thus increases the probability of default. Second, we find that a no bailout commitment is not sufficient to rule out the doom loop. A no bailout rule relieves the government from the fiscal burden to bail out banks and thus breaks the link between banks exposure and sovereign debt that drives the doom loop. But, assuming that banks' equity ratios remain unchanged, such a rule opens the door for a different type of self-fulfilling crises: If investors run on the bond and in doing so cause banks to fail, then this disrupts the intermediation system in the domestic economy and causes GDP to drop, weakening the government's incentives to repay its debt, justifying the initial run. The doom loop changes its nature but is sustained and becomes even worse. Third and most strikingly perhaps, we show that allowing banks to load up on sovereign bonds in sovereign crises is desirable. Such a policy might appear counterproductive at first sight: If banks exposure is the root of the doom loop, can it really help to increase this exposure exactly in times of crisis? It does because in times of distress banks' demand for domestic bonds supports the bond price without undermining the government's commitment to repay, as is the case for foreign debt. Thus the domestic banks' demand for bonds can resolve the distress and effectively rule out the doom loop.

In the second step we consider policies that rule out the doom loop but may nevertheless be undesirable. In particular we show that increasing banks equity ratio or rebalancing their portfolio away from domestic sovereign debt to a sufficient extent disables the doom loop – but only at the cost of eliminating the commitment device that local banks' exposure provides to

the government. Such policies thus undermine sovereign debt sustainability and make sovereign debt more costly, hence potentially reducing welfare. This argument also applies if the increase in the bank equity ratio is the result of a no bailout commitment as in Cooper and Nikolov (2018).³

Key for all these findings is that default incentives depend not only on the total amount of sovereign debt as commonly assumed, but also on who holds this debt. In the model defaulting on domestic banks is more costly than defaulting on foreign investors through the two channels mentioned before. Hence default incentives increase (decrease) in foreign-held (bank-held) debt. Note that for the results in step one the *temptation channel* – incentives to default on foreign debt are higher than on domestic debt – is enough. For the argument in step two however the *commitment channel* – banks' exposure increases the costs of default thus providing additional commitment – is key.

In a third step we consider two extensions. First, we consider a multi country extension of the model to show that bundling and tranching bonds of many countries in a union as suggested by Brunnermeier et al. (2017) in the context of the ESBies scheme does not resolve the doom loop either and can be detrimental for welfare. Second, while our baseline specification focuses on multiplicity of equilibria, in another extension we show that the same model also generates amplification. We illustrate this for the example of a news shock, that may resemble the current health crisis. We show that all the policy conclusions from before apply to that case: Policies that succeed to rule out (or not) multiple equilibria also are effective (or not) in ruling out amplification. Policies that rule out amplification by increasing equity or reducing domestic bond holdings come at a cost and may hence be undesirable.

Our model is highly stylized. Yet it highlights that a significant exposure of the domestic financial sector to the sovereign may not be as bad as previous theory suggests and that debt re-nationalization in bad times may be just what is necessary to avoid a market turmoil to develop into a full blown crisis. This argument is particularly relevant today, as government debt skyrockets across Europe and beyond. We thus provide an argument against policies that restrict the financial sectors exposure to domestic debt, which was prominently advocated by a group of German and French economists (Benassy-Quere et al., 2018). This proposal was soon criticized by Messori and Micossi (2018). Indeed, our model provides a formalization of their critique.

Related literature Our paper builds a bridge between two strands of literature. The first is the literature on the doom loop. Brunnermeier et al. (2016, 2017) and Cooper and Nikolov (2018) propose 3 period models that are very similar to ours. In their models multiplicity arises through the exact same mechanisms. Leonello (2017) shows that the doom loop can exist even if banks hold no explicit claims to the government on their balance sheets (bonds or debt) but enjoy government guarantees (deposit insurance, bailouts) and resolves the multiplicity of equilibria through global games. Acharay et al. (2014) and Farhi and Tirole (2016) provide a

³Cooper and Nikolov (2018) argue in favor of such a commitment not because it is effective per se, but because it incentivizes banks to self-insure against sovereign default by increasing their equity sufficiently.

slightly different notion of the doom loop. Instead of generating multiple equilibria, the doom loop serves as an amplifier, so that small fundamental changes can lead to large changes in the equilibrium. Our paper incorporates both notions of the doom loop and the policy conclusion apply to both as well. What distinguishes our model is that in these models the default incentives increase in total sovereign debt, while in our model the default incentives increase only in foreign held debt but not in bank held debt, which rather serves as a commitment device. This leads us to arrive to contrary policy conclusions.

The second strand regards the commitment role of domestic exposure to sovereign debt. This idea has been developed both in 3 period models (Balloch 2016, Basu 2010, Bolton and Jeanne 2011, Brutti 2011, Erce 2012, Gennaioli et al. 2014 and Mayer 2011) as well as in quantitative dynamic models (Boz et al. 2014, Balke 2018, Engler and Grosse Steffen 2016, Mallucci 2014, Sosa-Padilla 2018, Perez 2015, Thaler 2019). It is furthermore backed by empirical evidence. E.g. Gennaioli et al. (2014) show that sovereign default premia decrease in domestic bank's exposure to their sovereign.⁴ Relative to this literature we contribute by adding a notion of the doom loop.

2 Model

2.1 Setup

We consider a three period economy $t = 0, 1, 2$. The domestic economy is populated by a representative bank, a representative household and a benevolent government. The sovereign issues bonds at $t = 0$, which are held by the local bank and international creditors. At $t = 1$ a sunspot is realized and the sovereign decides if to bailout the bank. At $t = 2$ productivity is revealed, the government decides whether to repay or default, assets pay off and the household consumes.

Initial conditions The period 0 balance sheet of the bank and the government are exogenously given. The government has an initial level of outstanding bonds B_0 that are held by the local bank B_0^h or foreign creditors B_0^f , such that

$$B_0 = B_0^h + B_0^f$$

The initial assets of the bank are bonds B_0^h , loans to domestic firms L_0 and safe foreign bonds S_0 . Besides, it has deposit liabilities D_0 . All assets are promises to repay 1 unit in period $t = 2$. Each loan is backed by an investment project.

Period 1 At $t = 1$ the sunspot variable s is revealed, where $s \in \{n, p\}$ refers to normal times n or panic p . Sovereign debt markets open and the price of debt q_1 is determined. This affects

⁴For similar evidence see Sturzenegger and Zettelmeyer (2007), Acharay et al. (2014), Bolton and Jeanne (2011), Reinhart and Rogoff (2011b), and Balteanu and Erce (2017)

the bank's equity E_1

$$E_1 = q_1 B_0^h + L_0 + S_0 - D_0$$

If the price is low enough, the bank's equity may become negative. In that case the bank is insolvent and a fraction ϕ of the loans and the associated projects are destroyed. This captures the disruptions caused by a bank's insolvency. (Appendix C motivates this assumption by the possibility of bank runs.) Let L_1 denote the level of loans that remain at the end of period 1, if the banks are insolvent at the end of the period we have $L_1 = (1 - \phi)L_0$.

However, if banks get in trouble, the government can bailout the bank to avoid loan destruction. We assume that the bailout payment restores banks equity to 0.⁵ To finance the bailout it issues additional debt to international creditors ΔB_1^f , which also matures at $t = 2$. Denote the equilibrium price of debt in period 1 by q^{sb} , where s is the sunspot variable and $b \in \{0, 1\}$ is the indicator for the bailout decision. The transfer to banks is then given by $q^{s1} \Delta B_1^f$. The size of the transfer is

$$q^{s1} \Delta B_1^f = \max \left\{ D_0 - q^{s1} B_0^h - L_0 - S_0, 0 \right\}$$

where the max operator captures the fact that if the bank is solvent the required bailout is zero. Banks use these funds to invest in the safe foreign bond. The investment in the foreign bond in period 1 is given by $\Delta S_1 = q^{s1} \Delta B_1^f$ and total foreign bond holdings by $S_1 = S_0 + \Delta S_1$.

With a bailout the new level of outstanding debt held by foreign creditors corresponds to $B_1^f = B_0^f + \Delta B_1^f$. Foreign creditors are deep pocketed, competitive and risk neutral. Given a world interest rate of 1, their demand schedule requires the bond price to be equal to one minus the expected default probability at $t = 1$:

$$q^{sb} = 1 - \text{prob} \{ \text{default} \mid s, b \}$$

Period 2 At $t = 2$ productivity ω is revealed, then the government decides upon repayment ($d = 0$) or default ($d = 1$), production takes place, all assets pay off and the household consumes.

Bank equity in period 2, after the government announces its default decision, is:

$$E_2 = L_1 + S_1 - D_0 + B_0^h(1 - d)$$

No bailout is possible at this point. If banks are insolvent in period 2 or were insolvent and not bailed out in $t = 1$, again a fraction θ of the outstanding loans and associated projects gets destroyed and $L_2 = (1 - \theta)L_1$. Otherwise $L_2 = L_1$.⁶ Furthermore, if the government defaults the economy faces an output loss given by the fraction ϑ . Production then is given by

$$Y_2 = (1 - \vartheta d)\omega L_2$$

⁵Since the government has no other use for funds at this period and since a bailout smaller than this is not avoiding loan destruction, this is the optimal size of the bailout.

⁶This loss may be different than the loss in period 1, for example because the projects are already almost completed.

where ω is productivity and L_2 is the number of surviving loans.

We thus allow for two costs of sovereign default: One which depends on banks being solvent or not (θ), and one which does not (ϑ). So you may think of θ capturing the cost of sovereign default related to domestic financial turmoil and of ϑ as capturing all other reasons why default is costly. Both costs make defaulting costly to the government. Moreover, for the first part of the analysis below (sections 3 and 4) both costs are interchangeable, the only thing that matters is the total loss in case of bank insolvency and sovereign default $\Theta \equiv 1 - (1 - \theta)(1 - \vartheta)$.

The productivity level ω is a random variable with c.d.f $F(\omega)$ and support $[1/(1 - \vartheta), \bar{\omega}]$ where $\bar{\omega}$ is an arbitrary number $> 1/(1 - \vartheta)$. After production all assets pay off: The government taxes the household lump sum to repay its debt (if $d = 0$), the projects pay L_2 to the banks and $(\omega - 1)$ to the household, the bank pays back its deposits D_0 and distributes its equity to the household, who consumes everything. Consumption is hence equal to domestic net income:

$$C_2 = Y_2 + S_1 - (1 - d)B_1^f$$

Since the households are risk neutral, the benevolent government's objective is to maximize the expected value of C_2 . However the government has no commitment and hence solves a two stage problem. We summarize the optimization problem in Appendix A.

This concludes the setup of the model. We are now ready to define an equilibrium.

Definition. A subgame perfect equilibrium is given by bailout decisions $\{b^s\}_{s \in \{n,p\}}$, default policy functions $\{d^{sb}(\omega)\}_{s \in \{n,p\}, b \in \{0,1\}}$ and a price vector $\mathbf{q} = (q^{n0}, q^{n1}, q^{p0}, q^{p1})$ such that

1. The bailout decisions b^s and the default policy functions $d^{sb}(\omega)$ maximize expected consumption, taking as given the pricing vector \mathbf{q} .
2. The price q^{sb} corresponds to the expected repayment probability implied by the default policy function $d^{sb}(\omega)$ for each of the possible scenarios $\{n0, n1, p0, p1\}$.

We place a few restrictions on parameters.

Assumption 1. *The bank's deposit liabilities exceed its loans $D_0 > L_0 + S_0$.*

This assumption guarantees that the bank becomes insolvent if the bonds become worthless. It has two effects. First it opens up the possibility of multiple equilibria and thus the doom loop. Second, if $\theta > 0$ it means that the exposure of banks to sovereign debt acts as a commitment device. It ensures that the total cost of default is Θ , the sum of the direct costs ϑ and the costs due to bank fragility θ .

Assumption 2. *The banks holdings of sovereign debt are large enough to satisfy $B_0^h > (D_0 - L_0) \left(1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)\right)^{-1}$.*

This assumption ensures that banks hold enough bonds, such that an equilibrium exists where they are solvent in normal times, despite having less loans than deposits. It is hence the natural counterpart to the previous assumption.

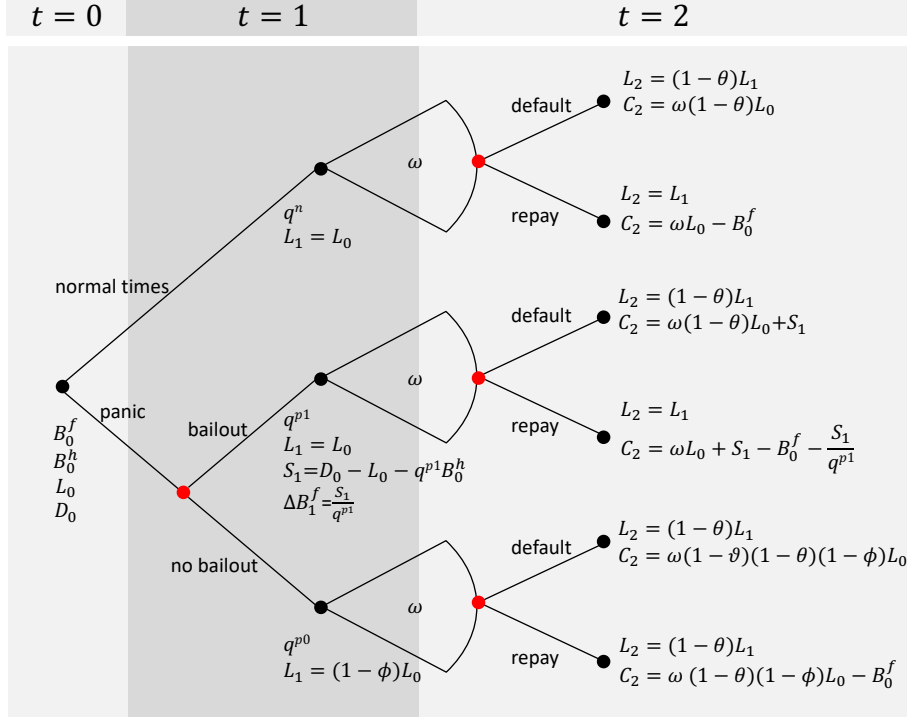


Figure 1: Timeline. This figure illustrates the timing of the model. Decision nodes are marked red.

Assumption 3. Banks hold no foreign bonds initially $S_0 = 0$.

This assumption is only made to simplify the analysis but is without loss of generality.

Figure 1 illustrates the model as a game in extensive form under this set of assumptions.

3 The doom loop

This model allows for several equilibria. We start by describing an equilibrium in which the bond price is unaffected by the sunspot and banks are solvent in period 1. Then we describe an equilibrium where banks are solvent in normal times ($s = n$), but insolvent in panic times ($s \in \{r, p\}$). In the second equilibrium, the allocation in normal times coincides with the allocation of the first equilibrium.

Proposition 1. An equilibrium where $q^{s,b}$ is the same for all n and b and banks are solvent in $t = 1$ (and hence require no bailout) exists and is unique. The equilibrium price and policy functions are given by

$$q = 1 - F(\omega^n)$$

$$d = \begin{cases} 1 & \omega < \omega^n \\ 0 & \omega \geq \omega^n \end{cases}$$

$$\text{where } \omega^n = \frac{1}{\Theta} \frac{B_0^f}{L_0}$$

The proof (Appendix B.1) is simple: It starts by solving the optimal repayment choice of the government in period 2 as a function of TFP ω , assuming that L_1, B_1^f, B_1^h are equal to their initial values. It then continues to show that this repayment policy implies a debt price q which is high enough to make banks solvent in period 1.

The optimal default policy is intuitive and in line with much of the literature on sovereign default: The government defaults for TFP below a certain threshold ω^n . The larger the foreign debt burden B_0^f , the larger the incentives to default. Conversely, the higher TFP ω and the greater the number of productive assets L_0 available at the last period, the lower the incentives of default. After all, the “punishment” for default is a proportional loss of the output produced by the productive asset. The equilibrium bond price q follows directly from the default decision. It is equal to the probability of default by the foreign lenders’ asset pricing condition.

Note that in this equilibrium sovereign debt can be sustained due to the combined default costs Θ .⁷ The nature of the default costs are irrelevant (θ vs. ϑ) since under assumptions 1 and 2 default also implies banks insolvency. In section 5 we analyze the case where banks have lower exposure such that default does not lead to bank insolvency.

Next we show that besides the fundamental equilibrium described above a sunspot equilibrium exists, where $q^{s,b}$ depends on s and banks are bailed out by the government in case of panic.

Equilibrium multiplicity and the doom loop.

Proposition 2. *For ϕ sufficiently large, a subgame-perfect sunspot equilibrium exists and is characterized by the following:*

- For $s = n$ (normal times) the policy and price functions are identical to before:⁸

$$b^n = 0 \tag{1}$$

$$q^n = 1 - F(\omega^n) \tag{2}$$

$$d^n(\omega) = \begin{cases} 1 & \omega < \omega^n \\ 0 & \omega \geq \omega^n \end{cases} \tag{3}$$

$$\text{where } \omega^n = \frac{1}{\Theta} \frac{B_0^f}{L_0} \tag{4}$$

- For $s = p$ (panic) the government decides to bailout banks $b^p = 1$. The price of debt $q^{p,1}$ and the default threshold $\omega^{p,1}$ are the solution to the system

$$q = 1 - F(\omega) \tag{5}$$

$$\omega = \omega^n + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0}{q} - B_0^h\right)}{L_0} \tag{6}$$

⁷By “being sustained” we mean that debt has a positive price at $t=1$ and would have a positive price in $t=0$ as well, although we do not model period 0.

⁸Where we omit the superscript b as when $s = n$ we have that the banks are solvent and require no bailout.

and the default policy is given by

$$d^{p,1}(\omega) = \begin{cases} 1 & \omega < \omega^{p,1} \\ 0 & \omega \geq \omega^{p,1} \end{cases} \quad (7)$$

- *Expected domestic consumption conditional on $s = p$ is lower than expected consumption conditional on $s = n$.*

- *In this subgame-perfect equilibrium, the former (on-equilibrium-path) panic price and policy are supported by the following (off-equilibrium) price and policy for $s = p$, conditional on no bailout*

$$q^{p,0} = 1 - F(\omega^{p,0}) \quad (8)$$

$$d^{p,0}(\omega) = \begin{cases} 1 & \omega < \omega^{p,0} \\ 0 & \omega \geq \omega^{p,0} \end{cases} \quad (9)$$

$$\text{where } \omega^{p,0} = \frac{\Theta}{\vartheta(1-\theta)(1-\phi)}\omega^n \quad (10)$$

The proof (Appendix B.2) formalizes a simple idea that we illustrate in Figure 2. Any equilibrium needs to be consistent with the lenders' asset pricing condition (equations (5) and (2)). The blue line in the figure shows the schedule defined by this condition. Since $F(\omega)$ is a c.d.f, $1 - F(\omega)$ has to be non-increasing – the higher the default threshold, the higher the default probability, the lower the price.

Furthermore, any equilibrium has to be consistent with the government's default policy. If banks are solvent in period 1, this policy can conveniently be characterized by the default threshold ω^n defined in equation (4). This threshold is independent of the debt price. In the figure it is shown as the vertical green line. The shaded area shows the levels of q for which the bank would be insolvent ($E_1 < 0$). If the green and the blue line intersect in the non-shaded solvency region the part of the equilibrium associated to $s = n$ exists.⁹

In the case that the bank is insolvent and needs to be bailed out in period 1, the relationship between the bond price and the default threshold given by the government's default policy is a little more complicated. It is defined by equation (6) and represented by the red schedule in the figure. In this case, the lower the bond price in period 1, the larger the equity shortfall and hence the larger the bailout. Furthermore, the lower the bond price the larger the amount of foreign debt that needs to be issued by the government to finance a bailout of a given size. Thus the lower the bond price, the larger the foreign debt. A larger debt burden in turn implies a higher default probability in period 2. Hence the red schedule is downward sloping. If the red and the blue lines intersect in the shaded (insolvency) region, the sunspot equilibrium exists.¹⁰

⁹Assumption 1 ensures that this intersection indeed lies in the solvency region.

¹⁰The existence of the intersection is guaranteed to exist and lie within the shaded region since (i) $F(\omega)$ has bounded support such that the blue schedule eventually reaches 0 (ii) the red schedule takes the value $q|_{E_1=0} < F(\omega^n)$ for ω^n , converges to 0 from above and is concave. For a generic cumulative distribution function F there may be more than one crossing. In that case we shall from now on focus on the crossing that

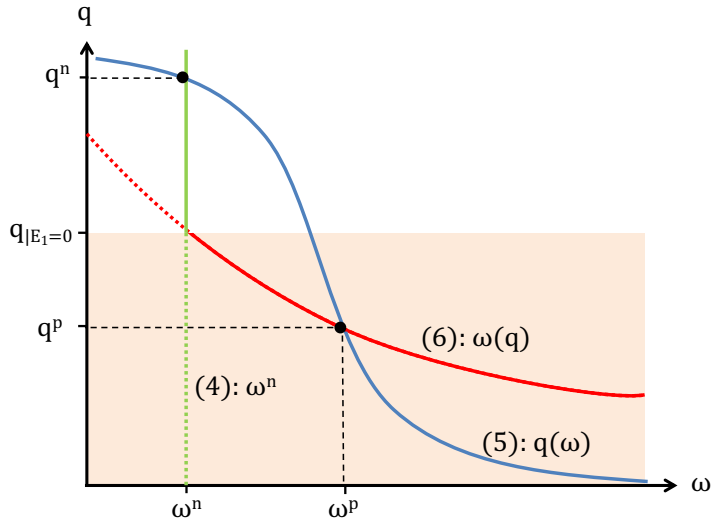


Figure 2: Equilibrium

Note that this is the infamous doom loop at play. Just as in Brunnermeier et al. (2016) and Cooper and Nikolov (2018), pessimistic expectations become self-fulfilling. If agents happen to coordinate on the lower bond price, banks become insolvent, forcing the government to increase its debt to finance a bailout, which makes it more likely that the government defaults later on (red curve). A higher default probability in turn justifies a lower bond price (blue curve). The pessimistic expectations are hence validated.

In this discussion of the sunspot equilibrium we focused on the case where bailout is the government's optimal choice. This is guaranteed by the condition that the loans losses in period 1 (ϕ) are a large enough (see Appendix B.2). This assumption is natural – if it were not satisfied, banks would never be bailed out and hence the doom loop would not exist. In section 4.3 we turn our attention to the bailout decision and the possibility of committing not to bailout banks.

For the bailout to be optimal it also has to be feasible. If creditors at $t = 1$ would expect a default with certainty and consequently the price of sovereign debt were zero, then the government could not bailout banks by issuing further debt – a bailout would be infeasible. However, as long as there is some probability mass above $\omega^{p,0}$ (the default threshold in case of no bailout) this panic is not self-fulfilling and consequently not an equilibrium. We focus on this case here.

Finally, note that the panic equilibrium, which we depict in the figure, is not locally stable under best response dynamics. However, as Cooper and Nikolov (2018) show it is easy to obtain a stable panic equilibrium price by putting adequate restrictions on the c.d.f. $F(\omega)$. All of our analysis goes through if we were to restrict our attention to such a stable equilibrium.

delivers the highest value of q^p .

4 Reducing exposure to break the doom loop?

Mutual exposure between the sovereign and the bank generates the doom loop. Thus breaking the doom loop may require reducing the banks exposure to sovereign debt or reducing the sovereign exposure to banks by no bailout rules. Indeed, as Brunnermeier et al. (2016) and Cooper and Nikolov (2018) show, in models where banks' exposure does not generate commitment to repay, both strategies can be successful at ruling out the bad equilibrium. The next two sections revisit these policies. Taking into account how the identity of the bond holders affects repayment incentives turns out to overturn these results or qualify them in important ways.

The fact that we arrive to such different results rests on one fundamental difference: While the doom loop literature assumes that the government's default decision is nonstrategic and driven by the total amount of debt, we assume it is strategic and consequently domestically and foreign held debt affect default incentives differently through the two channels mentioned in the introduction. First, in our model the temptation to default increases in foreign held debt relative to GDP (*temptation channel*). Domestically held debt does not increase the temptation to default, because repaying such debt is a zero sum game for the domestic economy. Second, if bank insolvency is costly in the second period $\theta > 0$, domestic debt additionally serves as a commitment device (*commitment channel*).

We start with a set of results that do not require the second difference. That is they go through even if $\theta = 0$ (assuming $\vartheta > 0$). In the subsequent section 5 we highlight the role of θ .

4.1 The role of the ex ante exposure

We start by considering a reduction in the exposure of banks ex ante. Policy proposals such as the ESBies proposal of Brunnermeier et al. (2016, 2017) suggest to reduce banks exposure by inducing them to diversify their sovereign holdings internationally. Thus we assume that, in period 0, banks sell a marginal amount of bonds to foreign investors at price q^n – that is a the price which would pertain if the sunspot was a 0 probability event – and use the proceeds to buy the safe foreign bond S_0 .¹¹ The following proposition summarizes the main result:

Proposition 3. *Lower exposure generates a lower price of sovereign debt and higher default probability in the normal state: $\frac{\partial q^n}{\partial B_0^h} > 0$.*

There exist productivity distributions $F(\omega)$ for which lower exposure generates a lower price of sovereign debt and higher default probability in the panic state: $\frac{\partial q^p}{\partial B_0^h} > 0$

The proof is in Appendix B.3. The intuition is simple: Since banks hold less domestic debt, the amount of debt held by foreigners increases. Since default incentives increase in the amount of foreign debt through the *temptation channel*, the default probability increases. This explains the fall in the price of debt in normal times q^n .

¹¹The case where banks use these proceeds to repay deposits and the household in turn invests these funds into a safe asset is isomorphic.

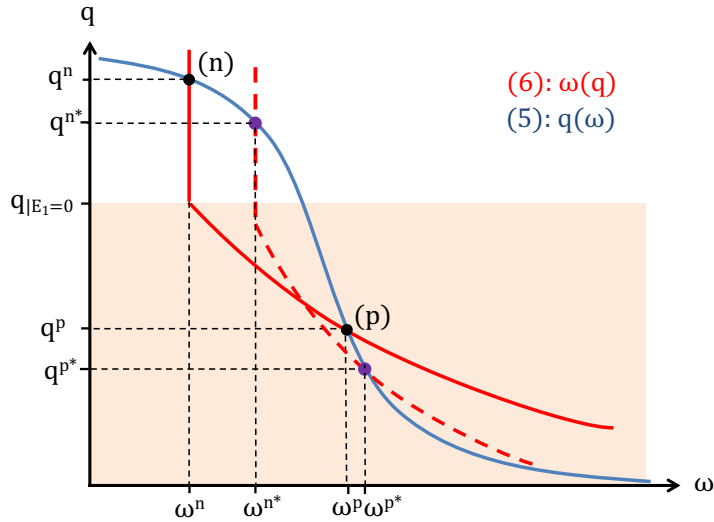


Figure 3: Equilibrium lowering B_0^h

The direction of the change in the price of debt in a panic is ambiguous. The higher debt held by foreigners increases the temptation to default and therefore the default probability for a bailout of a given volume. On the other hand, when exposure is reduced then a larger drop of the bond price is required to generate a bailout of the same volume. These two effects work in opposite directions and depending on F , the price q^p could fall or increase. The case for which the second effect dominates is illustrated in figure 3.

In sum, lowering the exposure makes normal times worse, does not eliminate the panic event, and can even make the panic worse. If the latter is the case or if the panic sunspot is sufficiently unlikely, ex-ante welfare hence drops.

4.2 The role of the ex post exposure

In the previous subsection we saw that reducing the banks ex ante exposure at the margin does not help to rule out multiple equilibria. But what about the banks exposure in period 1 – does reducing bank’s bond holdings in crisis times help? To analyze this question we first allow the government to bailout the banks by providing them with additional domestic debt. Then we show how this allocation can be decentralized when banks are allowed to re-optimize their portfolio in period 1.

4.2.1 Bailout by domestic bonds

Consider the model as in section 2.1 but now assume that in a bailout the government directly provides the bank with additional sovereign debt, instead of borrowing abroad to finance the safe investment. In this case a bailout no longer increases the foreign debt burden, which remains at its initial value B_0^f . Hence the benefit of default no longer increases in the size of

the bailout and thus the bond price $q^{p,1}$ has no reason to fall. Unlike before, the temptation channel is mute.¹² That means that the doom loop, which leads to multiplicity of equilibria in the baseline model, is no longer active and we can rule out the sunspot equilibrium.

Proposition 4. *When the bank is bailed out with domestic bonds, the sunspot equilibrium ceases to exist.*

Thus increasing the exposure of banks by issuing additional debt in times of self-fulfilling expectations driven crises is benign and, in this simple model, in fact rules out such crises altogether. This is so because such a bailout does not interact with the default incentives like a foreign debt financed bailout does. Models where only total debt determines repayment, but not the composition of bond holders, such as Brunnermeier et al. (2017), do not share this feature.

The result in Proposition 4 could also be decentralized by originally financing the bailout with international creditors and letting the newly issued debt to be traded in secondary markets, as we describe next.

4.2.2 Portfolio re-optimization and secondary markets

So far our bank was extremely passive; it had no decision to take. Now we extend the model to allow the bank to re-optimize its portfolio in period 1. Loans and deposits are assumed to be illiquid, but the bank can now choose whether to invest the bailout funds it receives (S_1) in domestic debt, subject to a regulatory maximal exposure constraint $q^{p,1}B_1^h \leq \bar{B}$.

Just as in the basic setup, the value of the bailout S_1 is given by

$$S_1 = \max \left\{ D_0 - L_0 - q^{p,1}B_0^h, 0 \right\}$$

and it is financed with the issuance of new debt such that

$$\Delta B_1 = \frac{1}{q^{p,1}} S_1$$

where the new debt is allocated to foreign investors or local banks through the secondary markets satisfying $\Delta B_1 = \Delta B_1^f + \Delta B_1^h$.

The bank operates under limited liability. Its objective hence is to maximize the expected non-negative part of equity in period 2 i.e. $\mathbb{E}[\max\{E_2, 0\}]$, subject to the budget constraint $S_1 + qB_0^h \leq S_2 + qB_1^h$, where S_2 denotes the bailout funds that the bank keeps in the safe asset. The bank is atomistic and thus takes all prices and the governments actions as given. Due to limited liability the bank always has an incentive to buy as much debt as possible, whenever the default probability is positive.^{13,14} It thus invests all the bailout funds into sovereign debt,

¹²The cost of default does not change with the bailout or the bond price either, because banks are bankrupt in case of default and solvent in case of repayment no matter how many bonds they got in the course of the bailout in period 1.

¹³By Assumption 2 a bailout is only necessary if the price is lower than 1 and hence if the default probability is positive.

¹⁴There are other reasons why banks may have a higher valuation of government bonds than foreign investors,

if the exposure limit permits, or up to the limit \bar{B} otherwise, and invests the rest of the bailout funds in the safe asset. Then we have that the change in sovereign debt holdings and the safe asset holdings by the bank are given by:

$$\begin{aligned}\Delta B_1^h &= \min \left\{ \bar{B}/q^{p,1} - B_0^h, \Delta B_1 \right\} \\ S_2 &= S_1 - q^{p,1} \Delta B_1^h\end{aligned}$$

It is immediately evident that for $\bar{B} \rightarrow \infty$ the equilibrium of this economy coincides with that of the economy analyzed before in Proposition 4, where the government bails out the bank with sovereign debt but no trading is possible after the bailout. On the other hand, for finite \bar{B} the economy could feature multiple equilibria. The following proposition characterizes for which values of \bar{B} this is the case.

Proposition 5. *If $\bar{B} \geq D_0 - L_0$ there is no sunspot equilibrium and the equilibrium corresponds to the equilibrium described in Proposition 1. If $\bar{B} < D_0 - L_0$ there is a sunspot equilibrium.*

This result establishes that to rule out the doom loop it is sufficient that the regulatory maximal exposure in the panic state \bar{B} is greater than the equity shortfall in case of default ($D_0 - L_0$). From Assumption 2 we have that in normal times the exposure is already higher than this threshold as $q^{1,n} B_0^h > D_0 - L_0$. Consequently, the doom loop only arises if banks face constraints that force them to lower bond holdings at market value sufficiently during panics. In such scenario, the panic is self-fulfilling as the lower exposure of banks weakens the incentives of the sovereign to repay. On the contrary, if banks in panic times are allowed to hold bonds up to a value not too much lower than in normal times, the doom loop ceases to exist.

4.3 Commitment not to bailout

If the necessity for a bailout creates the additional debt that raises the default probability and hence makes the bailout necessary in the first place, it may seem like a useful policy to rule out bailouts and thus rule out the sunspot equilibrium. However this need not necessarily be the case, since ruling out the bailout affects default incentives.

To show this, we analyze the model assuming that the government has credibly committed to never bailout the bank. Starting from Proposition 2, it is straightforward to analyze the possibility of a sunspot equilibrium under the no bailout commitment: Since our equilibrium definition is subgame perfection, it is sufficient to trim the branch of the decision tree related to the bailout (see also figure 1).

Indeed, it turns out that a sunspot equilibrium continues to exist. The following proposition states that and characterizes the sunspot equilibrium

Proposition 6. *Under a no bailout commitment a unique sunspot equilibrium exists. For $s = n$ the policy and price functions are as in proposition 2. For $s = p$, default is given by a threshold*

especially in times of crisis, such as: Regulatory reasons, financial repression, or non-atomistic behavior of banks.

strategy

$$d^p(\omega) = \begin{cases} 1 & \text{if } \omega \leq \frac{\Theta}{\vartheta(1-\phi)(1-\theta)}\omega^n \\ 0 & \text{else} \end{cases}$$

and the bond price is $q^p = 1 - F\left(\frac{\Theta}{\vartheta(1-\phi)(1-\theta)}\omega^n\right)$

The intuition for this result is simple: a banking crisis in period 1, if left unaddressed by the government, increases the default incentives because it weakens the economy in the repayment period. Hence even with a commitment not to bailout banks, which eliminates the traditional doom loop, an alternative form of the doom loop exists. What drives this result is the fact that output (i.e. the tax base) in period 2 depends on the banking sector's condition in period 1. Just like in the traditional doom loop foreign debt goes up as the government mops up the financial crisis, here GDP goes down because it doesn't. Since the ratio of foreign debt to default losses – which are proportional to GDP – determines the repayment decision through the *temptation channel*, the effect is a similar loop.

How does this sunspot equilibrium compare to the equilibrium with a bailout? Under the assumption of large enough loan losses upon insolvency in period 1 (ϕ) as in proposition 2,¹⁵ a no bailout commitment entails an ex ante welfare loss: Expected consumption conditional on $s = n$ remains unchanged, but expected consumption conditional on $s = p$ drops due to the losses associated to the unresolved banking crisis in period 1.

In sum, whenever the doom loop exists, the no bailout commitment is thus both time inconsistent – the government prefers a bailout in period 1 – and ex-ante suboptimal – the expected welfare associated to the sunspot equilibrium is strictly lower with a no bailout commitment than without.

This proposition makes clear that it is not enough to reduce the exposure of the government to the financial sector through implicit bailout guarantees to break the doom loop. However, the argument of Cooper and Nikolov (2018) in favor of a no bailout commitment is more subtle. We get to that in the next section.

5 The commitment value of exposure

In the previous section we showed that lowering bank's exposure ex ante doesn't help with the doom loop and that allowing debt re-nationalization during crises can destroy the loop. These results rested on the fact that only foreign debt *increases default incentives* – the *temptation channel*. In this section we highlight a second difference between domestic and foreign debt: Domestic debt *reduces default incentives* by increasing the costs of default due to the impact on banks balance sheets in period 2 – the *commitment channel*.

Before our analysis didn't depend on the nature of the default costs – all that mattered was $\Theta > 0$ as default always triggered both the exogenous output loss ϑ and the endogenous loss θ caused by bank insolvency. By contrast, now we compare situations in which default

¹⁵Recall that this assumption is natural, since it is necessary to generate the standard doom loop.

triggers bank insolvency to others when it does not. The consequences of bank insolvency (θ loans destroyed) only matter in the former situation and consequently the incentives to default are lower in the former. In other words, we now explore the role of bankbond holdings as a commitment device for the government.

Note that this commitment device is of discrete nature. If banks are sufficiently exposed default causes them to have negative equity, causing the additional default cost θ . Else, banks have positive equity no matter what and default does not cause the cost θ . This is a simplifying assumption that allowed us to highlight that the results in the previous section do not rely on θ . It could be relaxed by making default costs increasing in the equity shortfall.

The simple model employed so far is of limited use to explore the value of commitment: commitment mainly matters when the government issues debt and so far we simply took initial debt as given. We thus expand the model slightly by modeling the debt issuance in period 0 in a parsimonious way, thus adopting more of a long run perspective. All previous propositions continue to hold in the expanded model.

5.1 The model with initial debt issuance

We make 2 assumptions about period 0. First, we assume that in period 0 the government has to finance some expenditures X with debt. While their nature is irrelevant for the model, one may think of them as arising from a public health crisis. It thus issues the minimal amount of government debt B_0 necessary

$$X = q_0 B_0 \tag{11}$$

where q_0 is the price of debt in $t = 0$.¹⁶ This debt is bought by banks and foreign lenders. For simplicity we assume that investors price debt assigning zero probability to the panic $q_0 = q^n$.

Second, we assume that the household sector of the domestic economy has R own resources available. These resources are invested into the bank through deposits D_0 and equity E_0 . The bank invests there resources into loans L_0 , bonds $q_0 B_0^h$ and the safe asset S_0 . So we have that total resources equal the value of liabilities and assets of the bank:

$$R = D_0 + E_0 = q_0 B_0^h + L_0 + S_0 \tag{12}$$

The quantities X , R , L_0 , S_0 and E_0 are exogenously given parameters. For the policy experiments below we will vary the latter 2 parameters, thus changing the exposure of banks.

Taken together these two modifications imply that neither foreign lenders nor domestic banks make capital gains or losses when the bond price changes in period 0, as opposed to the simpler model where initial conditions were given in terms of asset holdings.¹⁷

¹⁶There could be more than one level of B_0 that satisfies this constraint as q_0 is itself a function of debt issuance. This is why we restrict our attention to the lowest value that satisfies the constraint. It is akin to require the government to be on the “good” side of the Laffer curve.

¹⁷Proposition 7 also holds without these extensions.

5.2 Reducing exposure enough to kill the doom loop

Since the doom loop arose from the bank's fragility, the doom loop can be avoided by reducing the bank's exposure sufficiently to make them immune to fluctuations in the value of sovereign debt. As Brunnermeier et al. (2016) show, this can be achieved by either raising the bank's equity ratio or by reducing their sovereign bond holdings. Indeed, Cooper and Nikolov (2018) use this insight to argue for a no bailout commitment: In their model such a commitment induces banks to self-insure and hold enough equity to never be in need of a bailout.¹⁸ We now revisit these policy proposals.

Compare our baseline economy from before, in which banks are exposed to fluctuations in the price of debt (assumption 1) i.e.

$$D_0 > L_0 + S_0$$

with an alternative economy where banks are not exposed because their safe assets cover their deposit liabilities

$$D_0^{ne} = L_0 + S_0^{ne} \tag{13}$$

where superscripts *ne* refers to no exposure. In this latter economy banks' equity is non-negative even if the bonds lose all their value. The doom loop thus disappears. Motivated by the above cited literature, we consider two variants of the no exposure economy. First, we consider the case that the banks adjust its liabilities structure, i.e. increases its equity ratio, all else equal, $\frac{E_0^{ne,E}}{D_0^{ne,E}} > \frac{E_0}{D_0}$, $D_0^{ne,E} + E_0^{ne,E} = R$. Second, we consider the case that bank adjusts its asset structure by buying less domestic debt and instead purchasing more of the safe asset, all else equal. $S_0^{ne,S} > S_0$, $q_0^{ne,S} B_0^{h,ne,S} < q_0 B_0^h$. The superscripts *E* and *S* refer to the two variants, no exposure by larger equity *E* or by higher holding of safe assets *S*. In both cases we keep the balance sheet size equal by 12.

The next proposition establishes that spreads are higher in the no exposure economy and that reaching no exposure by accumulating the safe asset increases spreads further.

Proposition 7. *In the no exposure economy, for ϑ large enough, there exists a unique equilibrium where banks are solvent in $t = 1$ (and hence require no bailout) and where the sunspot is irrelevant. The equilibrium price and policy functions are given by*

$$\begin{aligned} q &= 1 - F(\omega^{ne}) \\ d &= \begin{cases} 1 & \omega < \omega^{ne} \\ 0 & \omega \geq \omega^{ne} \end{cases} \\ \text{where } \omega^{ne} &= \frac{1}{\vartheta} \frac{B_0^{f,ne}}{L_0} \text{ and } B_0^{f,ne} \leq B_0^f \end{aligned} \tag{14}$$

¹⁸We do not model the bank's funding choice. However, since the no bailout commitment is irrelevant if the banks has enough equity, we can mimic their policy proposal by simply assuming that the equity ratio is high enough.

Assume $\theta > 0$. Bond prices are lower in the economies with no exposure than the economy with exposure $q_0^{ne} \leq q_0$. Prices are lower if exposure is avoided by substituting local sovereign debt for the safe asset than if exposure is avoided by increasing equity $q_0^{ne,S} \leq q_0^{ne,E}$. The equalities are strict if $f(\omega) > 0$.

To get at the intuition for this result it is useful to combine equations (11), (12) and (13) to get:

$$q_0^{ne,E} B_0^{f,ne,E} = q_0 B_0^f \quad (15)$$

$$q_0^{ne,S} B_0^{f,ne,S} = q_0 B_0^f + (D_0 - L_0) \quad (16)$$

As (15) shows, the amount of funds that the government has to raise from international creditors is unaffected by how much equity the bank has. However, when the bank has enough equity to be solvent even if the government defaults, the costs of default are lower ($\vartheta < \Theta$). Banks exposure no longer serves as a commitment device. Thus, the default threshold (14) is higher and the bond price lower $q_0^{ne,E} < q_0$ though the *commitment channel*. That in turn raises the sovereign debt necessary to finance the expenditures $X B_0^{f,ne,E} > B_0^f$. The latter makes default even more attractive through the *temptation channel* mentioned in the introduction, which amplifies the initial drop in the bond price.

Furthermore, as (16) shows, if exposure is adjusted by lowering the bank's holding of sovereign debt, then the government is required to raise more funds from the international creditors, given by the term $(D_0 - L_0)$. This implies that default becomes even more tempting through the *temptation channel*, over and above the loss of commitment already discussed for the previous case. This second effect is the same that we observed before in isolation in section 4.1, where the *commitment channel* was absent.

In sum, no matter how banks exposure is eliminated, the foreign debt burden increases and the cost of default decrease such that default becomes more likely. This is evident from comparing equations 14 and (4).

If there is enough probability mass that the productivity draw can fall in between the default thresholds 14 and (4) determining the bond prices in Proposition 7, then the ordering of prices translates into an ordering of welfare: the lower the price the lower expected consumption and thus welfare. The next proposition formalizes this claim:

Proposition 8. *Assume $\theta, \vartheta > 0$. If $f(\omega)\omega$ is non-decreasing in the interval $[0, \omega^{ne,S}]$ then welfare is lower in the economy with no exposure than in the economy with exposure in the no sunspot equilibrium (Proposition 1). If the exposure of banks is avoided by substituting local sovereign debt for the safe asset then welfare falls more than if exposure is avoided by increasing equity.*

The condition that $f(\omega)\omega$ is non-decreasing in the interval $[0, \omega^{ne,S}]$ is sufficient but not necessary. It guarantees that the probability of default in the economy with no exposure is

sufficiently higher such that the expected default costs increase despite of facing a lower cost in case of default. . Note that a uniform distribution satisfies this condition, as does a bell shaped distribution to the extent that default is a tail risk. Even if the density is decreasing over this interval the condition can be satisfied as long as $f'(\omega) > -f(\omega)/\omega$.

Note further that the existence of the “normal times” equilibrium is no longer guaranteed. If ϑ is too small – which we have ruled out in the proposition –, then the government simply doesn’t have enough (exogenous) commitment to finance its expenditures. That is the *commitment channel* would kick in so strongly that it would be unable to finance the expenditures X . This should certainly have some significant utility costs, but they are outside our simple model.

We have compared prices and welfare assigning the panic a probability of zero for simplicity. As the probability of the panic increases, prices and welfare in the exposure economy decrease and could fall below those of the no exposure economies. However, by continuity our results continue to hold if the probability of the panic occurring is small enough.

In sum, under certain conditions it is undesirable to kill the doom loop by ex-ante restricting banks exposure to the sovereign. This is true for both policies considered here: higher bank capital ratios and substitution towards safe assets, such as ESBies. Furthermore, the latter policy is more harmful than the first. By extension, a no-bailout commitment is also undesirable even if it causes banks to increase their capital ratios as in Cooper and Nikolov (2018).

Our findings conflict with the policy conclusions of Brunnermeier et al. (2016) and Cooper and Nikolov (2018). The reason for these different conclusions lies in the modeling of default incentives. In their models the incentives to default do not depend on the bank’s exposure, only on total debt. Hence ruling out the sunspot equilibrium comes free of any cost for public debt sustainability. On the contrary, in our model policies that kill the doom loop by reducing banks exposure affect default incentives negatively, reducing the commitment to repay and thus increasing default probabilities.¹⁹ At the same time muting the doom loop by allowing banks to act as lenders of last resort, as discussed in section 4.2, comes without these costs.

6 Extensions

6.1 Diversification, ESBies and the doom loop

One particular proposal to break the doom loop in response to the European debt crisis has been the creation of European Safe Bonds (ESBies) (see Brunnermeier et al. 2017). This proposal consists in creating a European safe asset by tranching a bundle of European sovereign debt. The senior tranche of this collateralized security would constitute a safe asset, and by restricting banks to hold only the senior tranche, as opposed to local sovereign debt, banks would become

¹⁹In Brunnermeier et al. and in Cooper and Nikolov’s baseline model, default is non-strategic and driven directly by an exogenous “tax capacity” process. Cooper and Nikolov consider strategic default in an extension, but the default incentives are modeled as independent of the bank’s balance sheet. Brunnermeier et al. (2016) do however not analyze welfare or claim desirability.

less exposed to the domestic sovereign and the doom loop would be avoided. If this policy is successful at creating a safe asset, then the doom loop would indeed disappear, as we have shown in the previous section. However, it would also lead to more debt being held by foreigners and thus the commitment value of banks' exposure would be lost. This would come at the costs of higher spreads in normal times and possibly cause welfare losses, as we have shown in the same section.

In this section we turn to another issue that can arise if this policy is implemented, and that renders it even less beneficial: If banks hold a diversified portfolio of sovereign debt, the doom loop may still be present, even if the bundle is tranching (ESBies). Just that the panic happens at the European level, and not at the level of a single country.

To make this point, we extend our single country model to a continuum of identical countries. We show that if the countries in isolation are exposed to the doom loop, then diversification and tranching do not remove that risk. The doom loop persists in the ESBies economy, and its mechanism is closely related to the original doom loop: If investors expect a surge in default rate among European sovereigns in $t=1$, then the value of the sovereign debt bundle falls, causing bank insolvency and the need for bailouts in all countries. Since bailouts are financed by additional debt issuance, more countries end up defaulting in $t=2$, validating the initial beliefs. The main two differences with the single country doom loop are: (i) a larger fraction of debt is held by foreigners; (ii) the default decision of a given country has no impact on the local financial system, thus banks' sovereign debt holdings no longer serve as a commitment device and the temptation to default is larger – these are again the two channels highlighted throughout the paper.

Consider a continuum of measure one of ex-ante identical countries, which are each characterized as in the baseline model. The total amount of debt issued by the continuum of countries is split between the amount held to create the asset bundle and the amount held directly by non-European foreign investors as follows

$$\int_0^1 B_0^i di = \mathcal{B}_0 + \int_0^1 B_0^{i,f}$$

where B_0^i is the debt issued at $t = 0$ by country i , \mathcal{B}_0 is bundle of sovereign debt and $B_0^{i,f}$ is the bonds issued by country i held by foreign investors. The bundle \mathcal{B}_0 is tranching into a senior tranche \mathcal{B}_0^s (the ESBies) and a junior tranche \mathcal{B}_0^j where the subordination level is given by ς . If $\varsigma = 0$ then there is no tranching, only diversification. We normalize the face value of a unit of the senior and the junior tranche equal to one, the total issuance of the senior and junior tranches are then $\mathcal{B}_0^s = (1 - \varsigma)\mathcal{B}_0$ and $\mathcal{B}_0^j = \varsigma\mathcal{B}_0$. Since we assume all countries to be identical, we focus on a symmetric equilibrium and from now on omit the superscript i .

Assume that banks can only hold the senior tranche of the bundle and, for simplicity, that the volume of the senior tranche \mathcal{B}_0^s is set to exactly satisfy European banks' demand for sovereign debt. The junior tranche will be held by non-European investors. Let Q_t^s be the price of the senior tranche and Q_t^j of the junior tranche. The constraint that total resources equal the value

of liabilities and assets of the bank now reads:

$$R = D_0 + E_0 = Q_0^s \mathcal{B}_0^s + L_0 \quad (17)$$

where, as before, the quantities R , L_0 and E_0 are exogenously given parameters.

The timing and decisions are as in the baseline model. However, now at $t = 1$ a sunspot $S = \{N, P\}$ is revealed that affects *all* the countries, where N refers to normal and P to Paneuropean panic. After S is revealed the governments have to decide if to bailout banks in case their equity becomes negative, in order to avoid the destruction of a fraction ϕ of loans. The bailout is financed with the issuance of additional sovereign debt sold to non-European investors. At $t = 2$ countries learn about their idiosyncratic productivity, which is iid distributed. Then each government decides if to repay the outstanding debt. Note that banks' solvency does not depend on the domestic default decision, since they hold the ESBies whose return is certain by the law of large numbers. Therefore only the exogenous output cost ϑ matter.

The following proposition shows that an equilibrium with the doom loop exists.

Proposition 9. *For ϕ sufficiently large, a subgame-perfect sunspot equilibrium exists and is characterized by the following:*

- For $S = N$ (normal times) no bailout is necessary ($b^N = 0$) and the policy and price functions are given by:²⁰

$$q^N = 1 - F(\omega^N) \quad (18)$$

$$Q^{N,s} = \min \left\{ 1, \frac{1 - F(\omega^N)}{1 - \varsigma} \right\} \quad (19)$$

$$Q^{N,j} = \max \left\{ 0, \frac{1 - F(\omega^N)}{1 - \varsigma} - 1 \right\} \quad (20)$$

$$d^N(\omega) = \begin{cases} 1 & \omega < \omega^N \\ 0 & \omega \geq \omega^N \end{cases} \quad (21)$$

$$\text{where } \omega^N = \frac{1}{\vartheta} \frac{B_0}{L_0} \quad (22)$$

- For $S = P$ (panic) the government decides to bailout banks $b^P = 1$. The price of debt $q^P, Q^{P,s}$ and the default threshold ω^P are the solution to the system

$$q = 1 - F(\omega) \quad (23)$$

$$\omega = \omega^N + \frac{1}{\vartheta} \frac{\left(\frac{D_0 - L_0}{q} - \frac{B_0^s}{1 - \varsigma} \right)}{L_0} \quad (24)$$

$$Q^s = \frac{1 - F(\omega)}{1 - \varsigma} \quad (25)$$

²⁰Where we omit the superscript b as when $s = n$ we have that the banks are solvent and require no bailout.

and $Q^{P,j}=0$ and the default policy is given by

$$d^P(\omega) = \begin{cases} 1 & \omega < \omega^P \\ 0 & \omega \geq \omega^P \end{cases} \quad (26)$$

The conditions for the existence of a panic equilibrium are the same as in Proposition 2. As long as banks are exposed to sovereign debt $D_0 > L_0$, bailouts are desirable and TFP ω has bounded support such an equilibrium exists. Diversification and tranching do not remove this equilibrium.

So how does the sunspot equilibrium without ESBies from Proposition 2 compare to the sunspot equilibrium with ESBies here? One evident difference is that the default threshold in normal times now depends on the ratio of total debt to exogenous default costs B_0/ϑ , as opposed to the ratio of foreign held debt and the sum of the exogenous costs of default and the financial disruption B_0^f/Θ . This outcome is essentially the same as in Proposition 7, just that here we assume that all debt is pooled and thus held by foreigners, whereas there we focused on the case where only a part is sold to foreigners. ESBies just provide a way to create the safe asset, which before we simply assumed to exist.²¹ Extending Propositions 7 and 8 it is straightforward to rank the bond prices q^n and q^N from Propositions 2 and 9 and the associated levels of welfare.²² Conditional on the normal state, the bond price is lower with ESBies than without and the default probability higher. Furthermore, welfare is lower with ESBies under the conditions on F in proposition 8.

The possibility of a panic $S = P$, and the associated higher default rates show that the introduction of ESBies do not rule out the doom loop. It nevertheless changes the nature of it, as now the panic affects all the countries at once. This result holds for any level of subordination. What a higher level of subordination does is to increase the spreads and consequently the fraction of defaults observed in the panic equilibrium. The higher the subordination, the greater the panic needs to be to make the banks insolvent and consequently the higher the cost of the bailouts. This translates into welfare (conditional on the panic) decreasing as subordination goes up.

Our results are in stark contrast with Brunnermeier et al. (2016, 2017). In those papers, diversification and tranching effectively rules out the doom loop. In particular for a high enough level of bank capitalization, the introduction of ESBies removes the risk of the doom loop. There are several differences between the two setups,²³ but the two key difference that

²¹Safe in the sense that there is no uncertainty about the payoff of the asset, even if the payoff is below the face value as some countries do default.

²²The ESBies economy in normal times (N) resembles the no exposure economy (ne, S) from Propositions 7 and 8 with the twist that $B_0^{h,ne,S} = 0$.

²³The default is not strategic in Brunnermeier et al. (2017) and the government repayment is restricted by the primary surplus that is a random variable with a binary distribution. Therefore the composition of debt holders is irrelevant for repayment as opposed to our setup, where repayment incentives depend on how much debt is held by local banks and foreign investors. The underlying structure to generate the doom loop is basically the same: there is a sunspot variable that generates debt repricing and if banks become insolvent then a fraction of the loans are destroyed.

explain why ESBies are effective in their setup are the following: First, they assume that bailouts are financed by issuing senior government debt that is paid back with certainty even if the remaining debt is partially defaulted upon and that is thus sold at face value. By contrast, in our model the government finances the bailout by issuing additional sovereign debt that has no preferential treatment with respect to previously issued debt and consequently is valued at market prices. Second, in our model default is strategic such that bond holders may end up getting nothing, while in theirs the government mechanically pays bond holders as much as it can given an exogenous tax capacity, such that they always get something. The first difference strengthens the strategic complementarities in our model, the second ensures the existence of a sunspot equilibrium with a nonzero bond price.

Since the panic is driven by a sunspot, our model of course remains silent about the probabilities of the panic in the ESBies and the baseline model. Furthermore, symmetry may not be a reasonable assumption, since ESBies may help less solvent countries to benefit from more solvent countries. Yet the results that normal times get worse (through the *temptation and commitment channels*) and that panics may still happen serve as warning.

6.2 Amplification

As Cooper and John (1988) show, strategic complementarities – such as the one between regarding the bond price which our model considers – typically lead not only to multiplicity of equilibria, but also to amplification. We illustrate this next and show that our policy results carry over, thus highlighting how our model relates to the doom loop’s amplifying role highlighted by Farhi and Tirole (2016) and Acharay et al. (2014).

To this end, we remove the sunspot shock and rule out the self fulfilling sunspot equilibrium and add a fundamental shock to agents expectations about the distribution of future productivity. That is, we replace the sunspot shock by a news shock in period 1. This shock may capture diverse negative developments, including the outbreak of a global pandemic that shifts down the distribution of expected GDP. We choose this shock to illustrate how the doom loop amplifies fundamental shocks, in this case a shock to future productivity. The nature of the shock is irrelevant. A contemporaneous shock to the asset quality of banks or the world interest rate, for example, would be amplified in the same way.

In period 1 a binary news shock $s \in \{n, r\}$ materializes: Either the distribution of productivity remains as before (F) or it changes to a stochastically dominated new distribution F^r . We assume that this distribution is sufficiently worse to cause a bond repricing that makes banks bankrupt. Note that this assumption about the bad state is the opposite of assumption 2, which guarantees that banks are solvent in the good state.

Assumption 4. \tilde{F} is such that banks are bankrupt at the bond price that would pertain if banks would survive period 1 independently of their solvency $\frac{1}{\Theta} \frac{B_0^f}{L_0}$: $B_0^h < \left(1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)\right)^{-1} (D_0 - L_0)$.

That is, we assume the opposite for the normal and the recession state: By assumption 2 banks are solvent in normal times, but by assumption 4 they are insolvent in recessions times.

The equilibrium now depends on the fundamental shock, but otherwise closely resembles the sunspot equilibrium in proposition 2:

Proposition 10. *For ϕ sufficiently large, a subgame-perfect equilibrium exists where banks are solvent in normal times, and where banks are bailed out else:*

- For $s = n$ (normal times) the policy and price functions are identical as in proposition 1
- For $s = r$ (recession) the government decides to bailout banks $b^r = 1$. The price of debt $q^{r,1}$ and the default threshold $\omega^{r,1}$ are the solution to the system

$$q = 1 - F^r(\omega) \quad (27)$$

$$\omega = \omega^n + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0}{q} - B_0^h\right)}{L_0} \quad (28)$$

with the highest value for q . The default policy is given by

$$d^{r,1}(\omega) = \begin{cases} 1 & \omega < \omega^{r,1} \\ 0 & \omega \geq \omega^{r,1} \end{cases} \quad (29)$$

- In this subgame-perfect equilibrium, the former (on-equilibrium-path) panic price and policy are supported by the following (off-equilibrium) price and policy for $s = p$, conditional on no bailout,

$$q^{r,0} = 1 - F^r(\omega^{r,0}) \quad (30)$$

$$d^{r,0}(\omega) = \begin{cases} 1 & \omega < \omega^{r,0} \\ 0 & \omega \geq \omega^{r,0} \end{cases} \quad (31)$$

$$\text{where } \omega^{r,0} = \frac{\omega^n}{1 - \phi} \quad (32)$$

Due to the doom loop the solution of the system (27)-(28) is not unique. Since we focus on amplification in this section, we restrict our attention to the equilibrium with the highest $q^{r,1}$.

To understand how the doom loop amplifies the news shock, consider an alternative version of the model where negative bank equity in period 1 is inconsequential ($\phi = 0$) such that the government would never bail out banks. In that case the equilibrium default threshold for ω would be always the same, no matter whether good or bad news arrive. The bond price would however reflect the relevant distribution of future productivity.

$$q^{r*} = 1 - F^r(\omega^*) \quad (33)$$

$$q^{n*} = 1 - F(\omega^*) \quad (34)$$

$$d^*(\omega) = \begin{cases} 1 & \omega < \omega^* \\ 0 & \omega \geq \omega^* \end{cases} \quad (35)$$

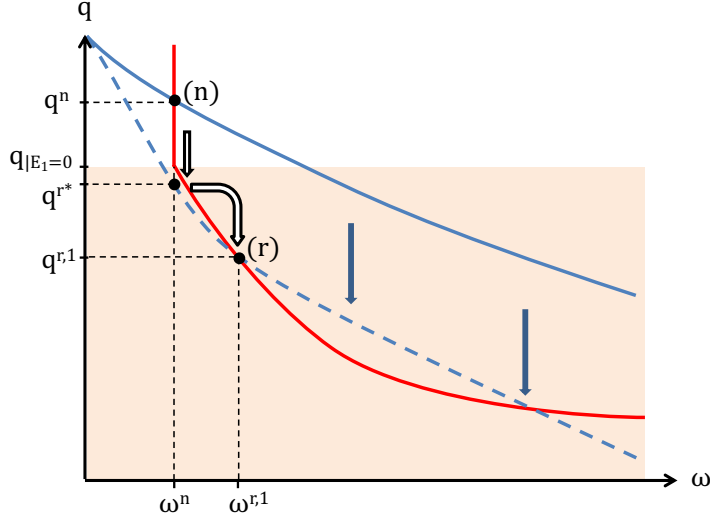


Figure 4: Amplification of a news shock

$$\text{where } \omega^* = \frac{1}{\Theta} \frac{B_0^f}{L_0} \quad (36)$$

Comparing the baseline and this alternative model, it is clear that nothing changes in good times. In bad times however the doom loop matters. Even if it is absent, the bond price drops in bad times, but if it is present, the drop is larger ($q^{r,0} < q^{r,*}$). That is, the doom loop amplifies the drop in bond prices caused by the fundamental shock. The same holds for the associated drop in welfare.

Figure 4 illustrates this graphically. When the bad state materializes, the bond price drops from q^n to $q^{r,*}$ in the absence of the doom loop. The doom loop then amplifies this initial drop and pushes the bond price further down to q^r .

Since it is the same strategic complementary that generates amplification and multiplicity, the results from the policy exercises in section 4 propositions 3 - 6 carry over to the case of amplification. Policies that (do not) help with multiplicity also (do not) help with amplification. Specifically: (i) reducing ex ante exposure makes normal times worse without removing amplification; (ii) domestic bailouts or (iii) secondary markets and a loose enough limit on bank bond holdings disable the doom loop and hence its amplifying effect; (iv) a no-bailout commitment does not disable the doom loop.

Furthermore, as in section 5, reducing the bank's exposure sufficiently by either increasing its equity ratio or decreasing its domestic bond holdings rules out the doom loop. This applies here too. However, propositions 7 and 8 apply as well, that is the success of these policies to rule out amplification has a cost in normal times: (v) reducing banks exposure to sovereign debt to the point that they are solvent regardless of the price of sovereign debt reduces the bond price in normal times and (vi) reduces welfare, conditional on normal times and hence if the recession state is sufficiently unlikely. We summarize these points:

Proposition 11. *Define amplification as a situation where $\omega^n < \omega^r$. Then:*

(i) *Lower exposure generates a lower price of sovereign debt and higher default probability in the normal state: $\frac{\partial q_0^n}{\partial B_0^h} |_{E_1^n = \bar{E}} > 0$. Furthermore there are productivity distributions $F(\omega)$ for which lower exposure generates a lower price of sovereign debt and higher default probability also in the recession states: $\frac{\partial q_0^s}{\partial B_0^h} |_{E_1^n = \bar{E}} > 0$, $s \in \{r, p\}$.*

(ii) *When the bank is bailed out with domestic bonds, there is no amplification.*

(iii) *With secondary markets, if $\bar{B} \geq D_0 - L_0$ there is no amplification and the equilibrium corresponds to the equilibrium described in Proposition 1. If $\bar{B} < D_0 - L_0$ there is amplification.*

(iv) *Under a no bailout commitment and maintaining the assumption of a high enough ϕ from Proposition 6, there is amplification*

(v) *Assume $\theta, \vartheta > 0$. Bond prices in normal times are lower in economies where banks are not exposed to sovereign debt than the economy with exposure $q_0^{ne} \leq q_0$. Prices are lower if banks are not exposed because they have substituted local sovereign debt for the safe asset than if they have increased equity $q_0^{ne,S} \leq q_0^{ne,E}$. The equalities are strict if $f(\omega) > 0$.*

(vi) *Assume $\theta, \vartheta > 0$. If $f(\omega)\omega$ is non-decreasing in the interval $[0, \omega^{ne,S}]$ then welfare is lower in the economy with no exposure than in the economy with exposure conditional on normal times. If banks are not exposed because they have substituted local sovereign debt for the safe asset then welfare falls more than if they have increased equity.*

7 Conclusion

Banks' exposure to sovereign debt give rise to the doom loop: A fall in the price of debt can require a bailout, which raises debt and hence the default probability, justifying the fall in the price of debt. However, the same exposure also provides commitment to the government, thus sustaining sovereign debt. This paper combines these two views to challenge two conclusions that can be derived from looking at the doom loop in isolation: (i) that banks exposure to their government should be reduced (ii) that it is desirable to commit not to bailout banks.

We question these policies along two dimensions. First we show that a marginal reduction in exposure and a no bailout commitment – if it doesn't lead to banks increasing their equity ratios – are ineffective at muting the doom loop and may make things worse, since they increase the incentives to default. Second, we show that an increase in the bank equity ratio – which may be a response to the no bailout commitment – or a reduction of bank bond holdings that is sufficiently large to mute the doom loop comes at a cost: Sovereign spreads in the no doom loop scenario rise as the commitment value of bank's exposure disappears and welfare drops. Rather, we argue that it is desirable that banks expand their exposure to public debt in times of sovereign distress, thus acting as lenders of last resort and breaking the doom loop.

These result may serve as a warning to the policy makers, which often express discomfort especially about banks high exposures to domestic sovereign debt. Maybe such exposure has more upsides than downsides after all. This is of particular relevance now that public debt is soaring due to the public health crisis.

While our model is no doubt stylized, it is straightforward to extend our analysis along several dimensions. First, parts of our analysis for simplicity assumed that the doom loop was a zero probability event. Yet by continuity our results would hold as long as it is sufficiently unlikely. Second, in our analysis banks' exposure did not affect the government's default cost *at the margin*. This allowed us to clearly separate between the effect negative and positive effects that foreign and domestically held debt have on repayment incentives. Allowing bank's exposure to also have a positive effect at the margin would strengthen our mechanism further.

Finally one caveat is in place. Our government is benevolent and maximizes national welfare. Thus there is no role for asset markets to discipline undesirable overspending by self-interested politicians, which might reduce the benefits of the additional commitment that bank's exposure provides.

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Appendix

A Problem of the government

The problem at $t = 2$ is

$$W_2^{sb}(\omega) = \max_{d \in \{0,1\}} \left[d \left(C_2^{sb,d} \right) + (1-d) C_2^{sb,r} \right]$$

where the $C_2^{sb,r}$ is the level of consumption for state s and bailout decision b if the government decides to repay. The levels of consumption for each eventuality are given by

$$C_2^{s0,r} = \begin{cases} \omega L_0 - B_0^f & \text{if } q^{s0} B_0^h + L_0 - D_0 \geq 0 \\ \omega(1-\phi)L_0 - B_0^f & \text{if } q^{s0} B_0^h + L_0 - D_0 < 0 \text{ and } B_0^h + (1-\phi)L_0 - D_0 \geq 0 \\ \omega(1-\phi)(1-\theta)L_0 - B_0^f & \text{else} \end{cases}$$

$$C_2^{s0,d} = \begin{cases} \omega(1-\vartheta)L_0 & \text{if } q^{s0} B_0^h + L_0 - D_0 \geq 0 \text{ and } L_0 - D_0 \geq 0 \\ \omega(1-\vartheta)(1-\theta)L_0 & \text{if } q^{s0} B_0^h + L_0 - D_0 \geq 0 \text{ and } L_0 - D_0 < 0 \\ \omega(1-\vartheta)(1-\phi)L_0 & \text{if } q^{s0} B_0^h + L_0 - D_0 < 0 \text{ and } L_0 - D_0 \geq 0 \\ \omega(1-\vartheta)(1-\phi)(1-\theta)L_0 & \text{else} \end{cases}$$

$$C_2^{s1,r} = \omega L_0 - B_0^f - \left(\frac{1}{q^{s1}} - 1 \right) \left(\max \{ D_0 - L_0 - q^{s1} B_0^h, 0 \} \right)$$

$$C_2^{s1,d} = \begin{cases} \omega(1-\vartheta)L_0 + \left(\max \{ D_0 - L_0 - q^{s1} B_0^h, 0 \} \right) & \text{if } L_0 - D_0 \geq 0 \\ \omega(1-\vartheta)(1-\theta)L_0 + \left(\max \{ D_0 - L_0 - q^{s1} B_0^h, 0 \} \right) & \text{if } L_0 - D_0 < 0 \end{cases}$$

and at $t=1$

$$W_1(s) = \max_{b \in \{0,1\}} \mathbb{E} \left[b W_2^{s1}(\omega) + (1-b) W_2^{s0}(\omega) \right]$$

B Proofs of propositions

B.1 Proposition 1

Proof. First we verify that the proposed equilibrium is indeed an equilibrium and then we show that no other equilibrium can exist.

Banks equity in $t = 1$ is given by

$$\begin{aligned} E_1 &= B_0^h q + L_0 - D_0 \\ \implies E_1 &= B_0^h \left(1 - F \left(\frac{1}{\Theta} \frac{B_0^f}{L_0} \right) \right) + L_0 - D_0 \end{aligned}$$

that from Assumption (2) is non negative. Therefore there is no bailout required and no loans are destroyed $L_1 = L_0$.

The default decision in $t = 2$ is the solution to

$$\max \left\{ \omega L_0 - B_0^f, \omega L_0 (1 - \theta)(1 - \vartheta) \right\} \quad (37)$$

where the first term is the consumption in case of repayment and the second is consumption in case of default. By Assumption 1, default makes the bank insolvent and consequently θ loans are destroyed and thus $L_2 = (1 - \theta)L_1$. Furthermore, sovereign default reduces output by ϑ .

The government defaults whenever

$$\begin{aligned} \omega L_0 - B_0^f &< \omega L_0 (1 - \theta)(1 - \vartheta) \\ \implies \omega &< \frac{1}{1 - (1 - \theta)(1 - \vartheta)} \frac{B_0^f}{L_0} = \frac{1}{\Theta} \frac{B_0^f}{L_0} \end{aligned}$$

and consequently the optimal policy is characterized by a threshold $\omega^n = \frac{1}{\Theta} \frac{B_0^f}{L_0}$. Given this default policy the price of sovereign debt in $t = 1$ is given by

$$\begin{aligned} q &= 1 - \text{prob}(\text{Default}) \\ \implies q &= 1 - F(\omega^n) \end{aligned}$$

so the proposed equilibrium is indeed an equilibrium.

Now suppose there is another $\tilde{q} \neq 1 - F(\omega^n)$ that supports another equilibrium where q does not depend on s and banks are solvent in $t = 1$. Since banks are solvent in $t = 1$ there is no bailout and the default decision is exactly the same as before. Note (37) does not depend on q . Therefore the threshold is also the same as before and given by ω^n and the equilibrium price has to satisfy

$$\tilde{q} = 1 - F(\omega^n)$$

a contradiction. □

B.2 Proposition 2

Proof. We have to verify that the price vector \mathbf{q} is indeed an equilibrium price vector. For that it has to be that the default decisions taking as given \mathbf{q} indeed imply a default probability for each possible entry that is equal to $1 - q^{sb}$, for each $s \in \{n, p\}$ and $b \in \{0, 1\}$.

Step 1: Normal times

Let's start with the case $s = n$. Analogous to the proof of proposition 1, if

$$q^n = 1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)$$

then banks are solvent, and there is no bailout in period 1. The government chooses whether or not to default maximizing

$$\max\left\{\omega L_0 - B_0^f, \omega L_0(1 - \theta)(1 - \vartheta)\right\}.$$

As above, the optimal policy is to default whenever the productivity draw is below $\omega^n = \frac{1}{\Theta} \frac{B_0^f}{L_0}$. This policy is hence consistent with the price q^n we started from.

Step 2: Panic with bailout

Next consider the case $s = p$. We first characterize the default decision in case there is a bailout. We guess and later verify that the price $q^{p,1}$ is positive. This condition is necessary to make a bailout feasible. In that case, if the government defaults the bank is insolvent, since it had 0 equity in period 1 when the bond was still having a positive value. The level of consumption in case of a default is

$$C_2^d = (1 - \theta)(1 - \vartheta)\omega L_0 + S_1$$

Where we have that θ of the loans are destroyed and S_1 is the bailout transfer. On the other hand, in case of repayment, banks equity E_2 is positive by Assumption 2 and the level of consumption is

$$C_2^r = \omega L_0 + S_1 - \left(B_0^f + \frac{1}{q^{p,1}} S_1\right)$$

The government decides to default if

$$\begin{aligned} C_2^d &> C_2^r \\ \implies (1 - \theta)(1 - \vartheta)\omega L_0 + S_1 &> \omega L_0 + S_1 - \left(B_0^f + \frac{1}{q^{p,1}} S_1\right) \\ \implies \omega &< \omega^n + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0}{q^{p,1}} - B_0^h\right)}{L_0} \end{aligned}$$

where we have just rearranged terms and using the definition of $S_1 = D_0 - L_0 - q^{p,1} B_0^h$ and $\omega^n = \frac{1}{\Theta} \frac{B_0^f}{L_0}$ and $\Theta = 1 - (1 - \theta)(1 - \vartheta)$. Therefore the optimal default policy is a threshold policy for a given bond price where the threshold is given by

$$\omega^{p,1} = \omega^n + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0}{q^{p,1}} - B_0^h\right)}{L_0} \quad (38)$$

for international creditors to break even ex-ante it has to be that

$$q^{p,1} = 1 - F(\omega^{p,1}) \quad (39)$$

so the values of q and ω that solve the system (38)-(39) are the equilibrium values. In the last step we show that this solution exists and features $q^{p,1} > 0$.

Step 3: Panic without bailout

Now consider the default decision if there was no bailout. In this case a fraction ϕ of loans is destroyed in $t = 1$ and a fraction θ in $t = 2$. Consumption in case of default hence is

$$C^d = (1 - \vartheta)(1 - \phi)(1 - \theta)\omega L_0$$

and in case of repayment

$$C^r = (1 - \phi)(1 - \theta)\omega L_0 - B_0^f$$

and consequently the default threshold satisfies

$$\begin{aligned} (1 - \phi)(1 - \theta)\omega L_0 - B_0^f &= (1 - \vartheta)(1 - \phi)(1 - \theta)\omega L_0 \\ \omega &= \frac{1}{\vartheta(1 - \phi)(1 - \theta)} \frac{B_0^f}{L_0} \\ \omega &= \frac{\Theta}{\vartheta(1 - \phi)(1 - \theta)} \omega^n \end{aligned}$$

from this it follows that the value of sovereign debt in case of no bailout is given by

$$q^{p,0} = 1 - F\left(\frac{\Theta}{\vartheta(1 - \phi)(1 - \theta)} \omega^n\right)$$

Step 4: Bailout decision

Welfare in case of no bailout corresponds to

$$\begin{aligned} W^{nb} &= \int_1^{\omega^{p0}} ((1 - \vartheta)(1 - \theta)(1 - \phi)\omega L_0) \partial F(\omega) + \int_{\omega^{p0}}^{\bar{\omega}} ((1 - \phi)(1 - \theta)\omega L_0 - B_0^f) \partial F(\omega) \\ \implies W^{nb} &= (1 - \theta)(1 - \phi)L_0 E\{\omega\} - (1 - F(\omega^{p0}))B_0^f - F(\omega^{p0})\vartheta(1 - \theta)(1 - \phi)L_0 \left(E\{\omega \mid \omega \leq \omega^{p0}\}\right) \end{aligned}$$

and using the envelope theorem we have that

$$\frac{\partial W^{nb}}{\partial \phi} = L_0(1 - \theta) \left(F(\omega^{p,0})\vartheta E\{\omega \mid \omega \leq \omega^{p0}\} - E\{\omega\}\right)$$

that is negative since $E\{\omega\} > E\{\omega \mid \omega \leq \omega^{p0}\}$ and $F(\omega^{p,0})\vartheta < 1$. Furthermore at the limit when $\phi = 1$ we have $W^{nb} = 0$. On the other hand, the welfare in case of a bailout is always positive and does not depend on ϕ . Therefore, there is a threshold for ϕ such that the bailout decision is optimal if ϕ is larger than the threshold.

Step 5: Existence of bailout solution

What remains to be shown is that the system (38)-(39) has a solution for $\omega^{p,1}, q^{p,1}$. From

equation (39) we can write ω as a function of q as follows

$$\omega = G_1(q) = F^{-1}(1 - q)$$

where $F^{-1}(\cdot)$ is the inverse function of F . Also define

$$G_2(q) = \omega^n + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0}{q} - B_0^h\right)}{L_0}$$

so we are left to show that there is a value of $q < \frac{D_0 - L_0}{B_0^h}$ (banks are insolvent) for which

$$G_1(q) = G_2(q)$$

First note that

$$G_2\left(\frac{D_0 - L_0}{B_0^h}\right) = \omega^n < G_1\left(\frac{D_0 - L_0}{B_0^h}\right)$$

This is so because: (i) $q = \frac{D_0 - L_0}{B_0^h}$ implies 0 equity, but q^n implies positive equity by assumption 2. Hence $\frac{D_0 - L_0}{B_0^h} < q^n$ (ii) $G_1(q^n) = \omega^n$ and (iii) G_1 is strictly decreasing. Hence $G_1\left(\frac{D_0 - L_0}{B_0^h}\right) > \omega^n$. So at $q = \frac{D_0 - L_0}{B_0^h}$ we have that $G_2 < G_1$.

Also as q tends to zero we have that $G_2(q)$ tends to ∞ , but given that ω is bounded, we have that $\min(G_1(0))$ is finite.

So summing up, i) G_1 and G_2 are continuous, ii) $\min(G_1(0))$ is finite and the limit $G_2(0) = \infty$; iii) at $q = \frac{D_0 - L_0}{B_0^h} > 0$ we have that $G_2(q) < G_1(q)$

i) ii) and iii) imply that G_1 and G_2 have to cross at least once in the interval $(0, \frac{D_0 - L_0}{B_0^h})$ and that at this intersection q is positive. Furthermore, the first time $G_1(q)$ and $G_2(q)$ cross, since $G_1(q)$ crosses from above we have that

$$\frac{\partial G_1(q)}{\partial q} < \frac{\partial G_2(q)}{\partial q} \tag{40}$$

Step 6: Welfare

Consumption conditional on the sunspot, productivity and the repayment decision are thus given by

$$\begin{aligned} C_2^{n1,r} &= \omega L_0 - B_0^f \\ C_2^{n1,d} &= \omega(1 - \vartheta)(1 - \theta)L_0 \\ C_2^{p1,r} &= \omega L_0 - B_0^f - \left(\frac{1}{q^{s1}} - 1\right) S_1 \\ C_2^{p1,d} &= \omega(1 - \vartheta)(1 - \theta)L_0 + S_1 \end{aligned}$$

And expected consumption, conditional on the sunspot, is

$$\begin{aligned}
\mathbb{E}_0 [C_2^{n1}] &= F(\omega^n) (\mathbb{E}_0 [\omega | \omega < \omega^n] (1 - \vartheta)(1 - \theta)L_0) + (1 - F(\omega^n)) (\mathbb{E}_0 [\omega | \omega > \omega^n] L_0 - B_0^f) \\
&= -F(\omega^n)\Theta L_0 \mathbb{E}_0 [\omega | \omega < \omega^n] - (1 - F(\omega^n)) B_0^f + \mathbb{E}_0 [\omega] L_0 \\
\mathbb{E}_0 [C_2^{p1}] &= F(\omega^{p,1}) (\mathbb{E}_0 [\omega | \omega < \omega^{p,1}] (1 - \vartheta)(1 - \theta)L_0 + S_1) + (1 - F(\omega^{p,1})) (\mathbb{E}_0 [\omega | \omega > \omega^{p,1}] L_0 - B_0^f) \\
&= F(\omega^{p,1}) (\mathbb{E}_0 [\omega | \omega < \omega^{p,1}] (1 - \vartheta)(1 - \theta)L_0) + (1 - F(\omega^{p,1})) (\mathbb{E}_0 [\omega | \omega > \omega^{p,1}] L_0 - B_0^f) + S_1 \\
&= -F(\omega^{p,1})\Theta L_0 \mathbb{E}_0 [\omega | \omega < \omega^{p,1}] - (1 - F(\omega^{p,1})) B_0^f + \mathbb{E}_0 [\omega] L_0
\end{aligned}$$

where in the last step we used the foreign lenders bond demand. Note that (41) and (42) both define the same function of welfare as a function of the default threshold, which we denote by $EC_1(\omega^{thres})$.

From step 1 we know that the repayment decision is such that C_2^{n1} maximized. Thus $\omega^n = \operatorname{argmax} EC_1(\omega^{thres})$. Since ω^n is unique and $\omega^{p,1} \neq \omega^n$ it must be that $\mathbb{E}_0 [C_2^{n1}] > \mathbb{E}_0 [C_2^{p1}]$. \square

B.3 Proposition 3

Proof. We analyze a marginal change in B_0^h on the price of debt as a function of the sunspot s . In particular, we consider a change in B_0^h where we keep total outstanding debt constant B_0 such that $\Delta B_0^h = -\Delta B_0^f$ and thus

$$\frac{\partial B_0^f}{\partial B_0^h} = -1$$

and where the bank uses all the proceeds from the sale of the domestic bond to purchase foreign bonds such that

$$\frac{\partial S_0}{\partial B_0^h} = - \left(q^n + \frac{\partial q^n}{\partial B_0^h} \right)$$

Normal times $s = n$

The default threshold and equilibrium price (ω^n, q^n) are given by the system

$$\begin{aligned}
\omega &= \frac{1}{\Theta} \frac{B_0^f}{L_0} \\
q &= 1 - F(\omega)
\end{aligned}$$

where we have that q^n satisfies

$$\begin{aligned}
q^n &= 1 - F \left(\frac{1}{\Theta} \frac{B_0^f}{L_0} \right) \\
\frac{\partial q^n}{\partial B_0^h} &= \frac{\partial \left(1 - F \left(\frac{1}{\Theta} \frac{B_0^f}{L_0} \right) \right)}{\partial B_0^h}
\end{aligned}$$

$$\begin{aligned}\frac{\partial q^n}{\partial B_0^h} &= -F'(\omega^n) \frac{1}{\Theta} \frac{1}{L_0} \frac{\partial B_0^f}{\partial B_0^h} \\ \frac{\partial q^n}{\partial B_0^h} &= F'(\omega^n) \frac{1}{\Theta} \frac{1}{L_0}\end{aligned}$$

and since $F'(\omega^n) \geq 0$ then we have that the price of debt at $s = n$ is not decreasing in B_0^h locally and if there is positive density at ω^n , then it is increasing in B_0^h .

Panic $s = p$

The default threshold and equilibrium price (ω^p, q^p) are given by the system

$$\begin{aligned}\omega &= \frac{1}{\Theta} \frac{B_0^f}{L_0} + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0 - S_0}{q} - B_0^h\right)}{L_0} \\ q &= 1 - F^p(\omega)\end{aligned}$$

where we have that q^p satisfies

$$q^p = 1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0} + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0 - S_0}{q^p} - B_0^h\right)}{L_0}\right)$$

By implicit differentiation

$$\begin{aligned}\frac{\partial q^p}{\partial B_0^h} &= \frac{\frac{1}{\Theta} \frac{1}{L_0} \left(\frac{q^n}{q^p} + \frac{\partial q_n}{\partial B_0^h} - 2\right)}{\left(\frac{1}{\Theta} \frac{1}{L_0} \frac{(D_0 - L_0 - S_0)}{(q^p)^2} - \frac{1}{F'(\omega^n)}\right)} \\ \frac{\partial q^p}{\partial B_0^h} \Big|_{S_0=0} &= \frac{\frac{1}{\Theta} \frac{1}{L_0} \left(\frac{q^n}{q^p} + \frac{\partial q_n}{\partial B_0^h} - 2\right)}{\left(\frac{1}{\Theta} \frac{1}{L_0} \frac{(D_0 - L_0)}{(q^p)^2} - \frac{1}{F'(\omega^n)}\right)}\end{aligned}$$

Now use the definitions from the previous proof, $G_1(q) = F^{-1}(1 - q)$ and $G_2(q) = \omega^n + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0}{q} - B_0^h\right)}{L_0}$ to rewrite

$$\frac{\partial q^p}{\partial B_0^h} \Big|_{S_0=0} = \frac{\frac{1}{\Theta} \frac{1}{L_0} \left(\frac{q^n}{q^p} + \frac{\partial q_n}{\partial B_0^h} - 2\right)}{(-G_2'(q) + G_1'(q))}$$

By equation (40) from the previous proof, the denominator is negative. Then the expression is positive if $\left(\frac{q^n}{q^p} + \frac{\partial q_n}{\partial B_0^h} - 2\right) < 0$. This condition can be satisfied by selecting F appropriately.

Welfare

Conditional on the normal state in period 1 consumption in period 2 as a function of ω and the default choice is equal to

$$C^{n,r} = \omega L_0 + S_0 - B_0^f$$

$$C_2^{n,d} = \omega(1 - \vartheta)(1 - \theta)L_0 + S_0$$

The $t = 0$ expectation of consumption conditional on the normal state occurring in period 1 is thus

$$\begin{aligned} \mathbb{E}_0 [C_2 | s = n] &= F(\omega^n) (\mathbb{E}_0 [\omega | \omega < \omega^n] (1 - \vartheta)(1 - \theta)L_0 + S_0) + (1 - F(\omega^n)) (\mathbb{E}_0 [\omega | \omega > \omega^n] L_0 + S_0 - B_0^f) \\ &= -F(\omega^n)\Theta L_0 \mathbb{E}_0 [\omega | \omega < \omega^n] - (1 - F(\omega^n)) B_0^f + S_0 + \mathbb{E}_0 [\omega] L_0 \\ &= -\Theta L_0 \int_0^{\omega^n} (\omega f(\omega) d\omega) - (1 - F(\omega^n)) B_0^f + S_0 + \mathbb{E}_0 [\omega] L_0 \\ &= -\Theta L_0 [\omega^n F(\omega^n) - FF(\omega^n)] - (1 - F(\omega^n)) B_0^f + S_0 + \mathbb{E}_0 [\omega] L_0 \\ &= -B_0^f F(\omega^n) + \Theta L_0 FF(\omega^n) - (1 - F(\omega^n)) B_0^f + S_0 + \mathbb{E}_0 [\omega] L_0 \\ &= \Theta L_0 FF(\omega^n) - B_0^f + S_0 + \mathbb{E}_0 [\omega] L_0 \end{aligned}$$

where in step 4 we used integration by parts and where FF is the anti-derivative of F and in step 5 we used the definition of ω^n .

Thus

$$\begin{aligned} \frac{\partial \mathbb{E}_0 [C_2 | s = n]}{\partial B_0^h} &= \frac{\partial FF(\omega^n)}{\partial B_0^h} \Theta L_0 + 1 - \left(q^n + \frac{\partial q_n}{\partial B_0^h} \right) \\ &= -F(\omega^n) + 1 - q^n - \frac{\partial q_n}{\partial B_0^h} \\ &= -\frac{\partial q_n}{\partial B_0^h} < 0 \end{aligned}$$

where in the last step we used the foreign lenders' bond pricing equation. Thus welfare conditional on normal times $s = n$ decreases in B_0^h . Thus unconditional welfare decreases if the probability of the sunspot is small enough. \square

B.4 Proposition 4

Proof. Assume that in the panic state the price falls such that the bank becomes insolvent

$$E_1 = q_1^p B_0^h + L_0 - D_0 < 0$$

the government bailout banks with the transfer of sovereign debt

$$\begin{aligned} q^p \Delta B_1^h &= -E_1 \\ \Delta B_1^h &= -\frac{1}{q^p} E_1 \end{aligned}$$

and there is no change in the debt holdings of foreign creditors $\Delta B_1^f = 0$.

The consumption in case of default is

$$c^d = (1 - \theta)(1 - \vartheta)\omega L_0$$

and the consumption in the case of repayment is

$$c^r = \omega L_0 - B_0^f$$

so the government defaults if

$$\begin{aligned} (1 - \theta)(1 - \vartheta)\omega L_0 &> \omega L_0 - (B_0^f) \\ \omega &< \frac{B_0^f}{\Theta L_0} \end{aligned} \quad (43)$$

and we have that the default threshold is the same as in normal times $\frac{B_0^f}{\Theta L_0}$ and consequently the price of debt in panic and normal times is the same. This violates the initial assumption that the bank is insolvent. The only equilibrium hence corresponds to the one presented in Proposition 1. \square

B.5 Proposition 5

Proof. If $s = p$ and the government decides to bailout banks, we have that equity before the bailout is

$$E_1 = q_1^p B_0^h + L_0 - D_0$$

and consequently the bailout transfer is given by

$$S_1 = \max \left\{ D_0 - L_0 - q^{p,1} B_0^h, 0 \right\}$$

Hence it has to do a debt issuance of

$$\Delta B_1 = \frac{1}{q_1^p} S_1 \quad (44)$$

Secondary markets open

Now the banks can bid for the newly issued bonds, using their risk-free bond. Banks can at most hold \bar{B} of their assets at market value in bonds. First consider the case in which the constraint binds. In this case the debt holding becomes $B_1^h = \frac{1}{q_1^p} \bar{B}$. The new debt that becomes foreign debt is then

$$\begin{aligned} \Delta B_1^f &= \frac{1}{q_1^p} S_1 - \left(\frac{1}{q^{p,1}} \bar{B} - B_0^h \right) \\ \Delta B_1^f &= \frac{1}{q_1^p} (D_0 - L_0 - \bar{B}) \end{aligned} \quad (45)$$

With these new levels of local and foreign debt, the consumption if the government defaults is

$$\begin{aligned} c^d &= (1 - \vartheta)(1 - \theta)\omega L_0 + S_1 - q^{p,1}\left(\frac{1}{q^{p,1}}\bar{B} - B_0^h\right) \\ &= (1 - \vartheta)(1 - \theta)\omega L_0 - (L_0 - D_0) - (\bar{B}) \end{aligned}$$

and under repayment is

$$\begin{aligned} c^r &= \omega L_0 + S_1 - q^{p,1}\left(\frac{1}{q^{p,1}}\bar{B} - B_0^h\right) - B_1^f \\ &= \omega L_0 + S_1 - q^{p,1}\left(\frac{1}{q^{p,1}}\bar{B} - B_0^h\right) - \left(B_0^f + \frac{1}{q^{p,1}}(D_0 - L_0 - \bar{B})\right) \end{aligned}$$

so the government defaults if $c^d > c^r$, that corresponds to

$$\begin{aligned} (1 - \vartheta)(1 - \theta)\omega L_0 + S_1 - q^{p,1}(\bar{B} - B_0^h) &> \omega L_0 + S_1 - q^{p,1}(\bar{B} - B_0^h) - \left(B_0^f + \frac{1}{q_1^p}(D_0 - L_0 - \bar{B})\right) \\ \omega &< \frac{B_0^f}{\Theta L_0} + \frac{\frac{1}{q_1^p}(D_0 - L_0 - \bar{B})}{(1 - \vartheta)L_0} \end{aligned}$$

and consequently the default decision follows a threshold strategy and the system of equation which solution correspond to $q^{p,1}$ and $\omega^{p,1}$ corresponds to

$$\begin{aligned} q^{p,1} &= 1 - F(\omega^{p,1}) \\ \omega^{p,1} &= \frac{B_0^f}{\Theta L_0} + \frac{\frac{1}{q^{p,1}}(D_0 - L_0 - \bar{B})}{\Theta L_0} \end{aligned}$$

Now consider the case where \bar{B} is not binding. In this case all the newly issued debt is acquired by the local banks and consequently $\Delta B_1^f = 0$. We see from equation (45) that this coincides with the previous case when $\bar{B} = D_0 - L_0$. Any larger value of \bar{B} is not binding as what restricts the purchase of sovereign debt by the bank is the constraint that the maximum amount of debt it can buy cannot have a value larger than the bailout received (S_1)

$$\Delta B_1^h = \frac{1}{q_1^p} S_1$$

and using equation (44) this implies that the maximum level of debt purchases are equal to the new debt issued ΔB_1 . This situation is then equivalent to and the price and default threshold are the solution to

$$\begin{aligned} q^{p,1} &= 1 - F(\omega^{p,1}) \\ \omega^{p,1} &= \frac{B_0^f}{\Theta L_0} \end{aligned}$$

The two cases can be summarized then for any \bar{B} by an equilibrium price $q^{p,1}$ and default threshold $\omega^{p,1}$ that solve the system

$$q^{p,1} = 1 - F(\omega^{p,1})$$

$$\omega^{p,1} = \frac{B_0^f}{\Theta L_0} + \frac{\max\left\{\frac{1}{q^{p,1}}(D_0 - L_0 - \bar{B}), 0\right\}}{\Theta L_0}$$

Hence, if $\bar{B} > D_0 - L_0$ the panic price is equal to the normal price and the sunspot equilibrium ceases to exist. \square

B.6 Proof Proposition 6

Proof. The proof of this statement follows directly from proposition 2: by ruling out bail-outs, the off equilibrium no-bailout branch becomes on-equilibrium. \square

B.7 Proof Proposition (7)

Proof. Consider two variations of the initial conditions of the baseline economy with initial debt issuance. In the first variation banks have a higher equity ratio, just high enough to ensure that they do not become insolvent when domestic sovereign bonds become worthless, i.e.

$$D_0^{ne,E} = L_0 \tag{46}$$

$$E_0^{ne,E} = R - D_0^{ne,E}$$

where the superscript ne, E denotes the initial conditions in this variation. The values of the other initial conditions (L_0, S_0) remain unchanged.

In the second variation, banks shift just enough of their assets from sovereign bonds to the safe asset to not become insolvent when domestic sovereign bonds become worthless, i.e.

$$L_0 + S_0^{ne,S} = D_0 \tag{47}$$

$$q_0^{ne} B_0^{h,ne,E} = E_0$$

where the superscript ne, S denotes the initial conditions in this variation. The values of the other initial conditions (L_0, D_0) remain unchanged.

The equilibrium in the ne economies is unique by the same argument as in proof 1. Following the results from Proposition (1) the price of debt in period 0 in an economy with no exposure is given by

$$q_0^{ne} = 1 - F(\omega^{ne}) \tag{48}$$

$$\omega^{ne} = \frac{1}{\vartheta} \frac{B_0^{f,ne}}{L_0}$$

For the price to be positive (such that the government can finance its expenditures and such that the bank can spend on bonds), it needs to hold that there is probability mass above ω^{ne} . Hence the restriction in the proposition “for ϑ large enough”.

To compare this price to the price q^n in the baseline specification we need to determine $B_0^{f,ne}$. $B_0^{f,ne}$ depends on how exactly we reduce the banks exposure in the ne economy. We consider each case in turn.

(i) Increase in equity

Step 1: Show that $B_0^{f,ne,E} \geq B_0^f$

We proof this statement by contradiction. Assume there is an equilibrium that satisfies $B_0^{f,ne,E} < B_0^f$. Then $q_0^{ne,E} = \left(1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right)\right)$ by ((48)).

Now, define the prices associated to this level of foreign debt in the baseline case (with lower bank equity) as $q'_0 \equiv \left(1 - F\left(\frac{1}{\Theta} \frac{B_0^{f,ne,E}}{L_0}\right)\right)$. Note that $q'_0 \geq q_0^{ne,E}$ due to $\Theta > \vartheta$ and that $q'_0 \geq q_0 = \left(1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)\right)$ due to $B_0^{f,ne,E} < B_0^f$.

Next, consider a deviation of the governments debt issuance in the baseline from B_0 to $B'_0 = B_0^{f,ne,E} + \frac{q_0}{q'_0} B_0^h$. Then banks will hold $B_0^{h'} = \frac{q_0}{q'_0} B_0^h$ by ((12)) and foreigners hold the rest $B_0^{f'} = B_0^{f,ne,E}$. The equilibrium price is then q'_0 by proposition 1. Furthermore, since $B_0^{f'} = B_0^{f,ne,E} < B_0^f$ and $B_0^{h'} \frac{q_0}{q'_0} \leq B_0^h$ we have that $B'_0 < B_0$.

Finally, compare the revenues raised by the governments debt issuance in the no exposure scenario and in the baseline scenario under the proposed deviation:

$$\begin{aligned} q_0^{ne,E} B_0^{ne,E} &= q_0^{ne,E} B_0^{f,ne,E} + q_0^{ne,E} B_0^{h,ne,E} \\ &= q_0^{ne,E} B_0^{f,ne,E} + R - S_0 - L_0 \\ q'_0 B'_0 &= q'_0 B_0^{f,ne,E} + q'_0 B_0^h \frac{q_0}{q'_0} \\ &= q'_0 B_0^{f,ne,E} + R - S_0 - L_0 \end{aligned}$$

where we used the fact that domestic resources are fixed ((12)) for the second and last equality. Since $q'_0 \geq q_0^{ne,E}$ we have $q'_0 B'_0 > q_0^{ne,E} B_0^{ne,E}$. Since $q_0^{ne,E} B_0^{ne,E} = X$ the deviation in the baseline must raise funds in excess of the needed quantities. $q'_0 B'_0 > X$. Thus, B_0 is not the lowest level of debt that guarantees that the government covers its expenditures, violating our assumption that the government issues the minimal level of debt necessary (i.e. is on the left of the debt laffer curve). Thus it cannot be that $B_0^{f,ne,E} < B_0^f$.

Step 2. Compare prices

The prices q_0 and $q_0^{ne,E}$ are given by

$$q_0 = 1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)$$

$$q_0^{ne,E} = 1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right)$$

Since $\Theta > \vartheta$ and $B_0^{f,ne,E} \geq B_0^f$ and since F is a cdf and thus nondecreasing we have that $q_0^{ne,E} \leq q_0$. Furthermore if $f(\omega)$ has any mass in the interval $\left[\frac{1}{\Theta} \frac{B_0^f}{L_0}, \frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right]$, then $q_0^{ne,E} < q_0$.

(ii) Increase in safe asset holdings

Step 1: Show that $B_0^{f,ne,E} > B_0^{f,ne,S}$

We proof this statement by contradiction. Assume there is an equilibrium that satisfies $B_0^{f,ne,S} \leq B_0^{f,ne,E}$. Then $q_0^{ne,S} = \left(1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}\right)\right) \geq q_0^{ne,E} = \left(1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right)\right)$.

Next, consider a deviation of the governments debt issuance in the ne, E case from $B_0^{ne,E}$ to $B'_0 = B_0^{f,ne,S} + \frac{q_0^{ne,E}}{q_0^{ne,S}} B_0^{h,ne,E}$. Guess that $q_0^{ne,S}$ is the associated equilibrium price. Then banks will hold $B_0^{h'} = \frac{q_0^{ne,E}}{q_0^{ne,S}} B_0^{h,ne,E}$ by ((12)) and foreigners hold the rest $B_0^{f'} = B_0^{f,ne,S}$. The equilibrium price is then $q_0^{ne,S}$, verifying our guess. Furthermore, since $B_0^{f'} = B_0^{f,ne,S} \leq B_0^{f,ne,E}$ and $B_0^{h'} = B_0^h \frac{q_0^{ne,E}}{q_0^{ne,S}} \leq B_0^h$ we have that $B'_0 \leq B_0^{ne,E}$.

Finally, compare the revenues raised by the governments debt issuance in the ne, E scenario

$$\begin{aligned} q_0^{ne,S} B_0^{ne,S} &= q_0^{ne,S} B_0^{f,ne,S} + q_0^{ne,S} B_0^{h,ne,S} \\ &= q_0^{ne,S} B_0^{f,ne,S} + R - S_0^{ne,S} - L_0 \\ &= q_0^{ne,S} B_0^{f,ne,S} + R - (D_0 - L_0) - L_0 \end{aligned}$$

to those raised under the proposed deviation

$$\begin{aligned} q_0^{ne,S} B'_0 &= q_0^{ne,S} B_0^{f,ne,S} + q_0^{ne,S} \frac{q_0^{ne,E}}{q_0^{ne,S}} B_0^{h,ne,E} \\ &= q_0^{ne,S} B_0^{f,ne,S} + R - S_0 - L_0 \\ &= q_0^{ne,S} B_0^{f,ne,S} + R - L_0 \end{aligned}$$

where we used the fact that domestic resources are fixed ((12)); that $S_0^{ne,S} = D_0 - L_0$; and that $S_0 = 0$ in the derivation.. Since $(D_0 - L_0) < 0$ by Assumption 1, we have that $q_0^{ne,S} B'_0 > q_0^{ne,S} B_0^{ne,S}$. Since $q_0^{ne,E} B_0^{ne,E} = X$ the deviation in the baseline must raise funds in excess of the expenditures $q_0^{ne,S} B'_0 > X$. Thus, B_0 is not the lowest level of debt that guarantees that the government covers its expenditures, violating our assumption that the government issues the minimal level of debt necessary (i.e. is on the left of the debt laffer curve). Thus it cannot be that $B_0^{f,ne,S} \leq B_0^{f,ne,E}$.

Step 2. Compare prices

The prices $q_0^{ne,S}$ and $q_0^{ne,E}$ are given by

$$q_0^{ne,S} = 1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}\right)$$

$$q_0^{ne,E} = 1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right)$$

Since $B_0^{f,ne,S} > B_0^{f,ne,E}$ and since F is a cdf and thus nondecreasing, we have that $q_0^{ne,E} \geq q_0^{ne,S}$. Furthermore if $f(\omega)$ has any mass in the interval $\left[\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}, \frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}\right]$ we have $q_0^{ne,E} > q_0^{ne,S}$. \square

B.8 Proof Proposition (8)

(i) Increase in equity

Consider the first variation from the previous proposition where banks have more equity, and compare it to the baseline economy. By ((11)) we know that the total revenues raised by the government's debt issuance is the same in both economies

$$q_0^{ne,E} (B_0^{f,ne,E} + B_0^{h,ne,E}) = q_0 (B_0^f + B_0^h) \quad (49)$$

and from ((12)) we can deduce that the bank's total assets are also unchanged

$$L_0 + q_0 B_0^h = L_0 + q_0^{ne} B_0^{h,ne} \quad (50)$$

Combining these two equations we get

$$q_0^{ne,E} B_0^{f,ne,E} = q_0 B_0^f \quad (51)$$

Next consider the consumption levels in the two economies. Consumption in the baseline economy with exposure – conditional on $s = n$ or the no sunspot equilibrium – is, in case of repayment r or default d , given by:

$$C^{n,r} = \omega L_0 - B_0^f \quad (52)$$

$$C_2^{n,d} = \omega(1 - \vartheta)(1 - \theta)L_0 \quad (53)$$

Expected consumption – conditional on $s = n$ or the no sunspot equilibrium – is thus

$$\begin{aligned} E(C|n) &= \left[1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)\right] \left[E\left(\omega \mid \omega > \frac{1}{\Theta} \frac{B_0^f}{L_0}\right) L_0 - B_0^f\right] + F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right) E\left(\omega \mid \omega < \frac{1}{\Theta} \frac{B_0^f}{L_0}\right) L_0(1 - \vartheta)(1 - \theta) \\ &= E(\omega) L_0 - \left[1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)\right] B_0^f - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right) E\left(\omega \mid \omega < \frac{1}{\Theta} \frac{B_0^f}{L_0}\right) L_0 \Theta \\ &= E(\omega) L_0 - \left[1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)\right] B_0^f - L_0 \Theta \int_0^{\frac{1}{\Theta} \frac{B_0^f}{L_0}} (\omega f(\omega) d\omega) \end{aligned}$$

Consumption in the economy with no exposure and higher equity is given by the same terms

as above (52) and (53). However, since the default costs are lower $\vartheta < \Theta$, the default threshold in the no exposure economy is different and expected consumption is thus

$$E(C|ne, E) = E(\omega) L_0 - \left[1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right) \right] B_0^{f,ne,E} - L_0 \vartheta \int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega)$$

we can use (51) and (48) to rewrite this as

$$E(C|ne, E) = E(\omega) L_0 - \left[1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right) \right] B_0^f - L_0 \vartheta \int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega)$$

The expected consumption and thus welfare in the economy with no exposure and higher equity if

$$E(C|n) > E(C|ne, E)$$

$$-B_0^f \left[1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right) \right] - L_0 \Theta \int_0^{\frac{1}{\Theta} \frac{B_0^f}{L_0}} (\omega f(\omega) d\omega) > - \left[1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right) \right] B_0^f - L_0 \vartheta \int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega)$$

We can simplify and rearrange this expression as follows, using in the last step again (51) and (48):

$$\begin{aligned} -L_0 \Theta \int_0^{\frac{1}{\Theta} \frac{B_0^f}{L_0}} (\omega f(\omega) d\omega) &> -L_0 \vartheta \int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega) \\ \frac{\int_0^{\frac{1}{\Theta} \frac{B_0^f}{L_0}} (\omega f(\omega) d\omega)}{\frac{B_0^f}{L_0 \Theta}} &< \frac{\int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega)}{\frac{B_0^f}{L_0 \vartheta}} \\ \frac{\int_0^{\frac{1}{\Theta} \frac{B_0^f}{L_0}} (\omega f(\omega) d\omega)}{\frac{B_0^f}{L_0 \Theta}} &< \frac{\int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega)}{\frac{B_0^{f,ne,E}}{L_0 \vartheta}} \frac{1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)}{1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right)} \end{aligned}$$

and since we have from the previous proposition that

$$\left(1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right) \right) \geq 1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right)$$

a sufficient but not necessary condition that guarantees $E(C|n) > E(C|ne, E)$ is

$$\frac{\int_0^{\frac{1}{\Theta} \frac{B_0^f}{L_0}} (\omega f(\omega) d\omega)}{\frac{B_0^f}{L_0 \Theta}} < \frac{\int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega)}{\frac{B_0^{f,ne,E}}{L_0 \vartheta}}$$

Note that each side of the inequality are the averages of $\omega f(\omega)$ within an interval. So since

$\frac{1}{\Theta} \frac{B_0^f}{L_0} < \frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}$ if we have that $\omega f(\omega)$ is not decreasing in $\left[0, \frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right]$ the inequality is satisfied. This condition implies $f'(\omega) \geq \frac{-f(\omega)}{\omega} \forall \omega \in \left[0, \frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right]$.

(ii) Increase in safe asset holdings

Consider the second variation from the previous proposition where banks have more safe assets and less bonds. By ((11)) we know that the total revenues raised by the government's debt issuance is the same in both economies

$$q_0^{ne,S} (B_0^{f,ne,S} + B_0^{h,ne,S}) = q_0 (B_0^f + B_0^h) \quad (54)$$

and from ((12)) we can deduce that the bank's total assets are also unchanged

$$L_0 + q_0 B_0^h = L_0 + q_0^{ne,S} B_0^{h,ne,S} + S_0^{ne,S} \quad (55)$$

Combining these two equations we get

$$q_0^{ne,S} B_0^{f,ne,S} - S_0^{ne,S} = q_0 B_0^f \quad (56)$$

and using (47) we get

$$q_0^{ne,S} B_0^{f,ne,S} - (D_0 - L_0) = q_0 B_0^f \quad (57)$$

Consumption in the economy with no exposure and higher safe asset holdings is given by the same terms as above (52) and (53) plus $S_0^{ne,S}$. Expected consumption is thus

$$E(C|ne, S) = E(\omega) L_0 - \left[1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}\right)\right] B_0^{f,ne,S} - L_0 \vartheta \int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}} (\omega f(\omega) d\omega) + S_0^{ne,S}$$

we can use (56) and (48) to rewrite this as

$$E(C|ne, S) = E(\omega) L_0 - \left[1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)\right] B_0^f - L_0 \vartheta \int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}} (\omega f(\omega) d\omega)$$

Next we compare welfare in the model variants with no exposure ne, E and ne, S . For the first variant to be preferable we need

$$\begin{aligned} E(C|ne, E) &> E(C|ne, S) \\ E(\omega) L_0 - \left[1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)\right] B_0^f - L_0 \vartheta \int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega) &> E(\omega) L_0 - \left[1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)\right] B_0^f - L_0 \vartheta \int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}} (\omega f(\omega) d\omega) \\ L_0 \vartheta \int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega) &< L_0 \vartheta \int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}} (\omega f(\omega) d\omega) \end{aligned}$$

using (57) and (51) we can write

$$\frac{\int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega)}{\frac{B_0^{f,ne,E}}{L_0 \vartheta}} < \frac{\int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}} (\omega f(\omega) d\omega)}{\frac{B_0^{f,ne,S}}{L_0 \vartheta}} \frac{1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}\right)}{1 - F\left(\frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}\right)} \frac{q_0 B_0^f + (D_0 - L_0)}{q_0 B_0^f}$$

The two rightmost fractions are larger than 1, (the first by the previous proposition, the second by Assumption 1). Thus a sufficient condition for $E(C|ne, E) > E(C|ne, S)$ is that

$$\frac{\int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0}} (\omega f(\omega) d\omega)}{\frac{B_0^{f,ne,E}}{L_0 \vartheta}} < \frac{\int_0^{\frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}} (\omega f(\omega) d\omega)}{\frac{B_0^{f,ne,S}}{L_0 \vartheta}}$$

So since $\frac{1}{\vartheta} \frac{B_0^{f,ne,E}}{L_0} < \frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}$ if we have that if $\omega f(\omega)$ is not decreasing in $\left(0, \frac{1}{\vartheta} \frac{B_0^{f,ne,S}}{L_0}\right)$ the inequality is satisfied.

B.9 Proof Proposition 9

Proof. Normal times: For $S = N$ we start by assuming that banks are solvent at $t = 1$ and then verify this claim once we find the equilibrium price.

If banks are solvent at $t = 1$, then no bailout is required and the consumption in case of repayment in $t = 2$ is given by

$$C_2^{r,N} = \omega L_0 + Q^{N,s} \mathcal{B}_0^S - B_0$$

and in case of default is

$$C_2^{d,N} = (1 - \vartheta) \omega L_0 + Q^{N,s} \mathcal{B}_0^S$$

where $Q^{N,s}$ is the price of the senior bond that coincides with the repayment fraction. The government will default if

$$\begin{aligned} C_2^{r,N} &\leq C_2^{d,N} \\ \omega L_0 + Q^{N,s} \mathcal{B}_0^S - B_0 &\leq (1 - \vartheta) \omega L_0 + Q^{N,s} \mathcal{B}_0^S \\ \omega &\leq \frac{1}{\vartheta} \frac{B_0}{L_0} \end{aligned}$$

then the default threshold is $\omega^N = \frac{1}{\vartheta} \frac{B_0}{L_0}$. From Assumption 2 we know that banks are solvent if the default threshold is $\omega^n = \frac{1}{\Theta} \frac{B_0^f}{L_0}$. Hence for ϑ not too much smaller than Θ banks are solvent at the default threshold ω^N too.

Panic: For $S = P$ we start by assuming banks are not solvent at $t = 1$ and then show that there is a debt price and default threshold that are an equilibrium and make banks insolvent.

If banks are insolvent at $t = 1$ and ϕ is large then a bailout is optimal and the funds needed

to save banks from insolvency are given by

$$Bailout = D_0 - L_0 - Q^{P,s} \mathcal{B}_0^s$$

where $Q^{P,s}$ is the price of the senior tranche in case of panic. To finance this bailout the government have to issue the amount of debt

$$\Delta B_1 = \frac{1}{q^P} (D_0 - L_0 - Q^{P,s} \mathcal{B}_0^s)$$

Then consumption in case of repayment is given by

$$C_2^{r,P} = \omega L_0 + Q^{P,s} \mathcal{B}_0^S - B_0 + \left(1 - \frac{1}{q^P}\right) (D_0 - L_0 - Q^{P,s} \mathcal{B}_0^s)$$

and in case of default is given by

$$C_2^{d,P} = (1 - \vartheta) \omega L_0 + Q^{P,s} \mathcal{B}_0^S + (D_0 - L_0 - Q^{P,s} \mathcal{B}_0^s)$$

the government then defaults if

$$C_2^{r,P} \leq C_2^{d,P}$$

$$\begin{aligned} \omega L_0 + Q^{P,s} \mathcal{B}_0^S - B_0 + \left(1 - \frac{1}{q^P}\right) (D_0 - L_0 - Q^{P,s} \mathcal{B}_0^s) &\leq (1 - \vartheta) \omega L_0 + Q^{P,s} \mathcal{B}_0^S + (D_0 - L_0 - Q^{P,s} \mathcal{B}_0^s) \\ \omega &\leq \frac{1}{\vartheta} \frac{B_0}{L_0} + \left(\frac{1}{q^P}\right) \frac{(D_0 - L_0 - Q^{P,s} \mathcal{B}_0^s)}{L_0} \\ \omega &\leq \omega^N + \left(\frac{1}{\vartheta q^P}\right) \frac{(D_0 - L_0 - Q^{P,s} \mathcal{B}_0^s)}{L_0} \end{aligned}$$

then the default threshold is given by

$$\omega^P = \omega^N + \frac{1}{\vartheta} \frac{\left(\frac{D_0 - L_0}{q^P} - \frac{Q^{P,s}}{q^P} \mathcal{B}_0^s\right)}{L_0}$$

and the equilibrium price of the bond taking as given that default threshold is

$$q^P = 1 - F(\omega^P)$$

The price of the senior tranche is 1 if the expected default rate is lower than the size of the senior tranche. Otherwise it falls proportionally in the default rate. It is given by

$$Q^{P,s} = \min \left\{ 1, \frac{1 - F(\omega^P)}{1 - \varsigma} \right\}$$

Since we started by assuming the banks equity is negative, $Q^{P,s} < 1$.

$$Q^{P,s} = \frac{1 - F(\omega^P)}{1 - \varsigma}$$

Then the equilibrium default threshold ω^P and the equilibrium price q^P are the values that solve the system

$$\begin{aligned}\omega &= \omega^N + \frac{1}{\vartheta} \left(\frac{D_0 - L_0}{q} - \frac{\mathcal{B}_0^s}{1 - \varsigma} \right) \\ q &= 1 - F(\omega)\end{aligned}\tag{58}$$

for ω and q . We can rewrite this system using two functions of q in terms of ω as

$$\begin{aligned}q_1(\omega) &\equiv \frac{D_0 - L_0}{\vartheta L_0 \omega - B_0^f + \frac{\mathcal{B}_0^s}{1 - \varsigma}} \\ q_2(\omega) &\equiv 1 - F(\omega)\end{aligned}$$

we have that: i) both functions are decreasing in ω and the $\lim_{\omega \rightarrow \infty} q_1(\omega) = 0$, ii) we have that at $\omega = \omega^N = \frac{1}{\vartheta} \frac{B_0}{L_0}$ the first function

$$q_1\left(\frac{1}{\vartheta} \frac{B_0}{L_0}\right) = \frac{D_0 - L_0}{\frac{\mathcal{B}_0^s}{1 - \varsigma}}$$

that is smaller than $q_2\left(\frac{1}{\vartheta} \frac{B_0}{L_0}\right)$. This follows from the fact that $q_2\left(\frac{1}{\vartheta} \frac{B_0}{L_0}\right)$ is the price of the bond in the normal state and implies positive equity while the price $q_1\left(\frac{1}{\vartheta} \frac{B_0}{L_0}\right)$ is the price level that makes equity equal to zero. (Recall that $E_1^P = L_0 - D_0 + \mathcal{B}_0^s Q^{P,s} = L_0 - D_0 + q^P \frac{\mathcal{B}_0^s}{1 - \varsigma}$, so $E_1^P\left(q_1\left(\frac{1}{\vartheta} \frac{B_0}{L_0}\right)\right) = 0$.) iii) We assume that the support of ω is bounded above and $F(\omega)$ is continuous. Then from i) ii) and iii) it follows that the system of equations 58 has at least one solution. \square

B.10 Proof Proposition 10

Proof. The proof follows directly from the proof of proposition 2. \square

B.11 Proof Proposition 11

Proof. Part (i) \square

We analyze a marginal change in B_0^h on the price of debt at each possible state realization. We consider a change in B_0^h where we keep total outstanding debt constant B_0 and also equity constant at the equilibrium price. By setting total debt constant we have that

$$\frac{\partial B_0^f}{\partial B_0^h} = -1$$

and by keeping equity constant at $q = q^n$ we have

$$\begin{aligned} \frac{\partial E_1^n}{\partial B_0^h} \Big|_{q^n} &= q^n - \frac{\partial D_0}{\partial B_0^h} = 0 \\ \implies \frac{\partial D_0}{\partial B_0^h} &= q^n \end{aligned}$$

Case i) $s = n$

The default threshold and equilibrium price (ω^n, q^n) are given by the system

$$\begin{aligned} \omega &= \frac{1}{\Theta} \frac{B_0^f}{L_0} \\ q &= 1 - F(\omega) \end{aligned}$$

where we have that q^n satisfies

$$\begin{aligned} q^n &= 1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right) \\ \frac{\partial q^n}{\partial B_0^h} &= \frac{\partial \left(1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0}\right)\right)}{\partial B_0^h} \\ \frac{\partial q^n}{\partial B_0^h} &= -F'(\omega^n) \frac{1}{\Theta} \frac{1}{L_0} \frac{\partial B_0^f}{\partial B_0^h} \\ \frac{\partial q^n}{\partial B_0^h} &= F'(\omega^n) \frac{1}{\Theta} \frac{1}{L_0} \end{aligned}$$

and since $F'(\omega^n) \geq 0$ then we have that the price of debt at $s = n$ is not decreasing in B_0^h locally and if there is positive density at ω^n , then it is increasing in B_0^h .

Case ii) $s = r$

The default threshold and equilibrium price (ω^r, q^r) are given by the system

$$\begin{aligned} \omega &= \frac{1}{\Theta} \frac{B_0^f}{L_0} + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0}{q} - B_0^h\right)}{L_0} \\ q &= 1 - F^r(\omega) \end{aligned}$$

where we have that q^r satisfies

$$\begin{aligned} q^r &= 1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0} + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0}{q^r} - B_0^h\right)}{L_0}\right) \\ \frac{\partial q^r}{\partial B_0^h} &= \frac{\partial \left(1 - F\left(\frac{1}{\Theta} \frac{B_0^f}{L_0} + \frac{1}{\Theta} \frac{\left(\frac{D_0 - L_0}{q^r} - B_0^h\right)}{L_0}\right)\right)}{\partial B_0^h} \end{aligned}$$

$$\begin{aligned}\frac{\partial q^r}{\partial B_0^h} &= -F'(\omega^n) \frac{1}{\Theta} \frac{1}{L_0} \left(\frac{\partial B_0^f}{\partial B_0^h} + \left(\frac{\frac{\partial D_0}{\partial B_0^h} q^r - \frac{\partial q^r}{\partial B_0^h} (D_0 - L_0)}{q^r} - 1 \right) \right) \\ \frac{\partial q^r}{\partial B_0^h} &= -F'(\omega^n) \frac{1}{\Theta} \frac{1}{L_0} \left(\frac{\frac{\partial D_0}{\partial B_0^h} q^r}{(q^r)^2} - 2 \right) + F'(\omega^n) \frac{1}{\Theta} \frac{1}{L_0} \left(\frac{\frac{\partial q^r}{\partial B_0^h} (D_0 - L_0)}{(q^r)^2} \right) \\ \frac{\partial q^r}{\partial B_0^h} &= \frac{\frac{1}{\Theta} \frac{1}{L_0} \left(\frac{q^n}{q^r} - 2 \right)}{\left(\frac{1}{\Theta} \frac{1}{L_0} \frac{(D_0 - L_0)}{(q^r)^2} - \frac{1}{F^r(\omega^n)} \right)}\end{aligned}$$

the denominator is negative as the curve $\omega(q)$ crosses $1 - F^r(\omega)$ from below. Then the expression is positive if $\left(\frac{q^n}{q^r} - 2\right) < 0$. This condition can be satisfied by selecting appropriately F^r .

Proof. Part (ii)

Following the first steps as in the proof of proposition 4, we have that the default threshold in a recession is the same as in normal times $\omega^n = \frac{B_0^f}{\theta L_0}$. Consequently the price of debt in panic and normal times is $q^n = 1 - F(\omega^n)$ and $q^r = 1 - F^r(\omega^n)$. The default threshold and bond price are hence the same as in the case that the bank did not need a bailout ($\phi = 0$). There is hence no amplification.

Part (iii)

Following the same steps as proposition 5, but replacing $F(\omega)$ by $F^r(\omega)$ we arrive to the conclusion that equilibrium price $q^{p,1}$ and default threshold $\omega^{p,1}$ solve the system

$$\begin{aligned}q &= 1 - F^r(\omega) \\ \omega &= \frac{B_0^f}{\Theta L_0} + \frac{\max\left\{\frac{1}{q} (D_0 - L_0 - \bar{B}), 0\right\}}{\Theta L_0}\end{aligned}$$

Hence if $D_0 - L_0 < \bar{B}$ then $\omega^{r,1} = \omega^n$, else $\omega^{r,1} > \omega^n$. Hence there is amplification if and only if $D_0 - L_0 > \bar{B}$.

Part (iv)

The proof of this statement follows directly from proposition 10: by ruling out bail-outs, the off equilibrium no-bailout branch becomes on-equilibrium. Therefore $\omega^{r,0} = \frac{\omega^n}{1-\phi} > \omega^n$ and we hence have amplification.

Part (v) and (vi)

The proofs of these statements follow directly from the proofs of propositions 7 and 8 \square

C Microfounding the equity requirement and loan destruction

In the text we simply *assumed* that negative equity causes a fraction of loans and the associated projects to be destroyed. This assumption can be micro-founded by bank runs.

Assume that the banks deposits are held by a continuum of depositors. In period 1 depositors can withdraw the funds they hold at the bank at face value. However, since the assets of the

bank are not liquid, the bank services such withdrawals by simply transferring its assets to the depositors. If the total volume of requested withdrawals exceeds the market value of the banks total assets, then the bank shuts down and distributes its assets among the requested withdrawals pro rata. Depositors that did not request a withdrawal get nothing. Depositors that withdrew their funds from the bank at $t=1$ simply hold the assets to maturity. However, since they are not as good at monitoring the existing assets as banks, a fraction ϕ of assets gets destroyed in $t = 1$ and then another fraction θ in $t = 2$. In the case of bonds the financial claim is simply canceled.²⁴ In the case of loans the underlying real asset disappears as well.

Now consider the choice of depositor i if to withdraw or not. Assume that the market value of banks assets exceeds the value of its liabilities. In that case the depositor can either withdraw, in which case he receives $(1 - \phi)(1 - \theta)D^i$ in period $t=2$. Or he can not withdraw, in which case he will be paid, in expectations, $D_0^i \int \min(1, \frac{(1-\phi)d\omega L_2}{D_0}) f(\omega) d\omega$ in $t=2$.²⁵ For ϕ large enough the depositor will prefer not to run. However, if the market value of banks assets is less than the value of its liabilities and all other depositors run than the depositor i gets nothing in case he does not run. Thus he will run too.

Thus a run equilibrium exists whenever banks have negative equity, but not when equity is positive. To micro-found the requirement that equity must be positive for loans not to be destroyed, we can thus simply focus on the case where depositors always coordinate to run whenever a run is possible.

The same story applies to period 2. Here it is enough if $\theta > 0$.

²⁴This assumption could be given up. We maintain it to simplify the algebra.

²⁵This expression can be simplified even further by assuming that there is deposit insurance.