Debt crises, fast and slow*

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Abstract

With the Covid-19 pandemic, public debt around the world is rising to unprecedented heights in peacetime, creating vulnerability to belief-driven sovereign risk crises. We revisit the mechanisms in strategic-default and debt-limit models, showing that slow-moving and rollover (fast) crises are pervasive in both. Sovereigns may lose market access even when, in a "good equilibrium", they would be able to borrow risk free. Long debt maturities may/may not shield countries from this adverse scenario. In a sunspot equilibrium, the threat of belief-driven crises is not enough to motivate deleveraging in a recession: governments will keep borrowing, gambling on economic recovery.

Key words: Sovereign default, Self-fulfilling crises, Expectations, Debt sustainability

JEL classification: E43, E62, H50, H63

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"The assessment of the Governing Council is that we are in [...] a "bad equilibrium", namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." Mario Draghi, ECB Press Conference, Transcript from the Q&A, September 6 2012

1 Introduction

From the Global Financial Crisis to the end of 2019, the average public debt to GDP ratio in advanced countries rose from below 80 percent to well above 100 percent. In 2020, the global distress of the COVID-19 pandemic is sparking a further hike in this ratio, raising a host of issues in financial and macroeconomic stability. The academic and policy literature has long reflected on the possibility that countries with relatively high debt face disruptive belief-driven turmoil in the sovereign bond market, increasing borrowing costs up to feeding unsustainable debt dynamics, or even resulting in sudden stops and rollover crises. As highlighted by the quote introducing this text, the exercise pursued by this literature is far from a theoretical *curiosum*. The turmoil in the sovereign debt market in the euro area after 2010 provides a vivid and striking example of the widespread disruption this type of crises can produce even among advanced countries.

In this paper, we reconsider the logic of belief-driven sovereign debt crises, showing that similar mechanisms can generate both slow-moving but unsustainable accumulation of public sector liabilities, and rollover (fast) crises—sudden stop in market financing. Our analysis is best appreciated in relation to two leading strands of the literature of debt sustainability. One draws on the seminal contribution by Calvo (1988), inspired by the persistent and unstable inflation experienced by Brazil in the 1980s at times of large fiscal imbalances. Multiple equilibria emerge, featuring the possibility of belief driven outright default or inflationary debt debasement, as investors price government bonds depending on their expectations of future debt paths. The anticipation of a steep path leading to default causes interest rates to rise; higher borrowing costs in turn accelerate debt accumulation; with a high and growing stock of debt, default occurs as soon as the economy is hit by a sufficiently negative shock, validating investors' pessimistic expectations. In view of this dynamic, in notable recent work Lorenzoni and Werning (2019) dub these crises "slow-moving". The turmoil in the sovereign debt market in the euro area after 2010 provides a striking example of the disruption this type of crises can produce.[1] The other strand of literature draws on the seminal work by Cole and Kehoe (2000), inspired by the experience of Mexico in the mid 1990s. As the government auctions off its debt, agents may coordinate their belief on an imminent default and decide not to participate in the auction—a rollover crisis then forces the government to default, again validating agents' expectations.[2] Since the switch across equilibrium coincides with a sudden loss of market access, these crises are "fast". Technically, the two models differ in a key assumption concerning the timeline along which the government sets how much bonds to issue, and investors set their price. At a deeper level, they shed light on different ways in which a liquidity crisis may occur.

Our analysis shows that, in addition to slow-moving debt crises, fast ones, in the form of rollover crises, are pervasive in models adopting a dynamic setting after Calvo (1988), independently of the mechanism highlighted by Cole and Kehoe (2000). The reason is insightful from both a theoretical and a practical vantage point. When the regime of investors' expectations turns from optimistic to pessimistic, the switch in investors' expectations causes the debt tolerance threshold of the government—the debt level above which default becomes the dominant action conditional on weak fundamentals—to fall. Depending on the initial level of debt, it is possible that the best response of the government to deteriorating expectations does not deliver enough adjustment for its financing need to be satisfied at any finite equilibrium interest rates. As the market anticipates this, the government loses market access and the country faces a debt rollover crisis.

As our baseline, we set up a dynamic model where a discretionary government optimizes its fiscal policy, deciding in each period whether to adjust primary surpluses or default. In doing so, we borrow the setting of Conesa and Kehoe (2017), except that the timing of the auction is that of Calvo (1988)[3]—hence the model

[1]The Calvo model has also been revived by Corsetti and Dedola (2016), in the context of an analysis of monetary backstops to government debt inspired by the launch of the Outright Monetary Transactions programme by the European Central Bank in 2012.

[2]Recent contributions in this class include Bocola and Dovis (2019), Conesa and Kehoe (2017), Bianchi and Mondragon (2018) and Corsetti et al. (2020), among others.

[3]See the discussion in Lorenzoni and Werning (2019), Corsetti and Dedola (2016) and Ayres et al. (2018)

abstracts from rollover crises à la Cole and Kehoe (2000). Starting from this environment, we also flesh out a version of our model close to Lorenzoni and Werning (2019). In this version, the decision to default is dictated by the debt limit implied by the country's natural budget constraint, evaluated at the maximum acceptable primary surplus the economy can deliver across time and circumstances. We refer to the two versions of our model as, respectively, the "strategic default" model (our baseline), and the "debt-limit" model. In our baseline, debt is sustainable as long as, conditional on both the state of the economy *and* expectations, the value of repaying is above the value of defaulting. In the debt-limit model, instead, the government tries to avoid reputation—it acts as if the output cost of default were so high that a crisis would compromise its capacity to sustain essential spending. It may nonetheless end up in a crisis when shocks and market dynamics push its stock of debt above the limit implied by the country's natural budget constraint.

In either version of the model, the debt tolerance threshold depends on the regime of expectations. We find that slow crises are possible at intermediate levels of initial debt—in the numerical example using our baseline, for debt levels between 59% and 121% of GDP. When debt is in this range, investors' pessimism translates into high borrowing costs that in turn ignite a slow-moving debt crisis: the hike in interest rates accelerates the dynamic of debt accumulation, and leads to default when the economic conditions worsen or if they fail to improve early enough. Fast crises are possible at higher levels of debt—in our example, between 121% and 204%. With debt in this range, as investors become pessimistic, they anticipate that the government will not be willing/able to undertake the required adjustment to make a good compromise on its liabilities. In other words, the market comes to believe that there is no positive bond price that satisfies the government's (re-optimized) financing need and the pricing equilibrium conditions. When this happens, the country loses market access and the government simultaneously defaults. In this sense, fast crises take the form of rollover crises—analytically different from the one studied by Cole and Kehoe (2000), but clearly appealing to the same economic logic.

We show that lengthening the maturity of government debt is not necessarily effective in ruling out equilibrium multiplicity leading to slow-moving debt crises—these remain pervasive in both our baseline and the debt-limit model. However, longer maturities may rule out fast rollover crises. In the debt-limit scenario, this is the case when the debt maturity that is not too short *and* the probability of a recovery is non-negligible. In our baseline, the parameter restrictions for ruling out fast crises are more stringent.

Last but not least, when an arbitrarily small probability of switching from the good to the bad equilibrium is internalized by both investors and policymakers, the government may have an incentive to keep debt below the relevant debt thresholds hence deleverage even during a recession. Different from Cole and Kehoe (2000), however, in our model(s) the incentive for the policymakers to deleverage drives fiscal policy only over a relative small range of debt levels around the debt thresholds. In a recession, for debt levels sufficiently higher than the thresholds, the consumption smoothing motive dominates governments' optimal policy, causing deficits and debt accumulation—i.e., gambling on the recovery.

From a policy perspective, our analysis has at least two implications for debt sustainability analysis and policy design. First, estimates of debt tolerance thresholds are a crucial input in assessing the extent to which a country can steer away from default. In this paper, we stress that these thresholds are not only contingent on the current and future state of the economy and/or preferences of the policymakers. They are also sensitive to investors' expectations. This introduces a further complication in debt sustainability analysis, hardly discussed by the existing literature.

Second, our analysis suggests that pervasive rollover risk may not provide enough welfare-incentive for implementing (even optimally smoothed) debt reduction strategies. We stress that this result is obtained independently of political economy considerations, with policymakers modelled as short-sighted or self-interested. In our framework, even a forward-looking benevolent government will generally find it optimal to raise debt in a recession, smoothing consumption at the cost of keeping the country in a state of vulnerability to self-fulfilling crises. This result may strengthen the case for an international compact, offering countries a combination of liquidity assistance and official loans favoring economically and politically acceptable policies of deleveraging. In both respects, our analysis contributes to ongoing policy and academic work reconsidering the modalities and structure of official lending and assistance by international organizations.

The Literature. This paper draws on the seminal contributions by Calvo (1988) and Conesa and Kehoe (2017), in turn related to Cole and Kehoe (2000). Calvo (1988) introduced the feedback loop between self-fulfilling expectations and debt burdens in a two-period model, where the government's financing need is taken as given, and the price and quantity of bonds are jointly determined in equilibrium. Self-fulfilling expectations of default generate market "runs" that manifest themselves in a surge in the interest rate charged by investors to the government—but no rollover crisis is modelled in the same context. Conversely, Conesa and Kehoe (2017) focus on liquidity crises whereby the market may suddenly become unwilling to roll over government's debt in anticipation of a default. In our paper we aim at reconsidering the nature and dynamic of rollover crises—we do so specifying a model on the style of Calvo (1988) model, but adopting a dynamic setting using the same environment as Conesa and Kehoe (2017), except for the specification of auctions underlying their view of rollover crises.

It is virtually impossible to provide a fair account of the rich literature on debt crises that has contributed to these two paradigms, directly and indirectly.[4] In Eaton and Gersovitz (1981), Arellano (2008) and Conesa and Kehoe (2017), the government commits to bond issuance before the market sets bond prices. This, together with an assumption restricting equilibrium pricing if no rollover crisis materializes, allows these authors to abstract from the multiplicity problem stressed by Calvo (1988). Also assuming that the government presets bond issuance, Auclert and Rognlie (2016) expand on Eaton and Gersovitz (1981) and discuss conditions such that a unique equilibrium exists. In a recent contribution, Aguiar and Amador (2020) show that short-term bonds are efficient as the government faces correct incentives to reduce default risk, in line with Aguiar et al. (2019). Relative to these papers, we find it reasonable that the government adjusts its policies based on the equilibrium bond prices it observes in the market. The government chooses bond issuance and primary deficits (the government financing need) as a function of the (equilibrium) interest rate.

[4] A partial list includes Alesina et al. (1989), Yue (2010), Chatterjee and Eyigungor (2012), Mendoza and Yue (2012), Arellano and Ramanarayanan (2012), Lizarazo (2013), the Handbook chapter by Aguiar and Amador (2014), Collard et al. (2015), Tirole (2015), and Bianchi et al. (2018)

Lorenzoni and Werning (2019) reconsiders Calvo (1988) in a dynamic setting, stressing that the increase in the sovereign's borrowing costs driven by self-fulfilling expectations of default leads a country to accumulate debt slowly but relentlessly over time. As the debt stock rises, at some point default occurs unless the conditions of the economy improve sufficiently. Ayres et al. (2018) adopts a framework similar to Arellano (2008) but for the timing assumption, to investigate the likelihood that a country becomes vulnerable to belief-driven crises. Also drawing on Calvo (1988), Corsetti and Dedola (2016) and Bacchetta et al. (2018) write monetary models and discuss how the central bank can backstop government debt, i.e. eliminate self-fulfilling crises by using, respectively, either unconventional (balance sheet) policy, or conventional (inflation) policy.[5]

Several paper have been developing the model with rollover crises of Cole and Kehoe (2000), into new directions. By way of example, Bocola and Dovis (2019) characterize how the maturity of sovereign debt can be structured to respond to rollover risk and fundamental risk. Aguiar et al. (2019) consider a variant of rollover crises modelling uncertainty in social utility upon defaulting. Chamon (2007) elaborates on the idea that the way in which sovereign bonds are underwritten and offered by investment banks may guard a country against rollover crises. Rollover crises are also modelled and discussed by Giavazzi and Pagano (1989), Alesina et al. (1992) and Cole and Kehoe (1996).

Finally, in writing this paper we draw extensively on previous work on debt bailout, especially on Corsetti et al. (2017), which introduces official lending in a Conesa and Kehoe (2017) framework, but also on Corsetti et al. (2020), Conesa and Kehoe (2014), Roch and Uhlig (2018) and Marin (2017).

This paper is organized as follows. Section 2 lays out the model similar to Conesa and Kehoe (2017) but with a different timing assumption. For our baseline, Section 3 discusses equilibrium multiplicity with different type of crises, while Section 4 presents a calibrated numerical example and offers a discussion of the model equilibria with long and short-term debt. Section 5 analyzes whether the perceived threat of a belief-driven crisis would prevent a government from running deficits during recessions. Section 6 reconsiders the analysis in a debt-limit framework. Sec-

[5]See also Aguiar et al. (2013).

tion 7 carries out sensitivity analysis focusing on debt maturity and probabilities of recovery. Section 8 concludes.

2 Model

In this section we specify our dynamic model of debt sustainability and default. The environment draws on Conesa and Kehoe (2017), except that we set the timing of investors' and the government decisions in the style of models after Calvo (1988), such that a government with a given financing need would not be able to set the amount of bonds to be issued before investors set bond price.

The state of the economy in every period, $s = (B, z_{-1}, a)$, is (i) the level of government debt owed to the risk neutral investors B, (ii) whether default has occurred in the past $z_{-1} = 0$ or not $z_{-1} = 1$, (iii) whether the economy is in a recession $a = 0$ or not $a = 1$. As in Conesa and Kehoe (2017), the country's GDP is

$$y(a, z) = A^{1-a}Z^{1-z}\bar{y}$$

with $A \leq 1$, and $Z < 1$. The parameter A denotes the business cycle: a recession occurs when $A < 1$. When the government defaults, the penalty is a permanent drop in productivity by the factor Z.

The economy starts out with $a_0 = 0$ and $z = 1$. From period 1, the economy recovers with probability $p < 1$ and once recovered, it never falls back to recession again. If the government defaults, it stays at the state of default $z = 0$ forever.

The government issues non-contingent bonds to risk neutral investors. As is customary after Hatchondo and Martinez (2009), we model maturity of government bonds as follows. Bonds have geometrically decreasing coupons: a bond issued at t pays the sequence of coupons

$$\kappa, (1-\delta)\kappa, (1-\delta)^2\kappa...$$

where $\delta \in [0, 1]$. Hence, assuming risk neutral investors whose discount factor is β, the price of a default-free bond is:

$$q = \frac{\beta\kappa}{1-\beta(1-\delta)}$$

To normalize bond prices, it is convenient to set $\kappa = 1-\beta+\beta\delta$ so that the price of a default-free bond is β. The parameter δ indexes the maturity of debt, where $\delta = 0$ corresponds to the case of "consols" (or perpetuities) and $\delta = 1$ corresponds to the case of short-term bonds. Note that a bond issued at $t-m$ is equivalent to $(1-\delta)^m$ bonds issued at t. Hence, the stock of outstanding bonds can be summarized by a single state variable, B.

As in Conesa and Kehoe (2017), the tax rate θ is constant for the government, so that sovereign tax revenue is $\theta y(a,z)$. We denote the endogenous government spending as g, and we stipulate that there is some critical expenditure level $\bar{g}$, below which the normal functioning of the state becomes problematic. The government's budget constraint is given by

$$zq(B',s)(B'-(1-\delta)B) = g + z\kappa B - \theta y(a,z)$$

where the right hand side defines the (endogenous) Gross Financing Need (GFN) of the government. As regards the timing of debt issuance, we follow Calvo (1988), see Figure 1.[6]

1. The aggregate state $s = \{(B, z_{-1}, a)\}$ is known. Each of a continuum of measure one risk neutral investors set bond price $q(b',s)$.

2. The government decides to default or repay, which determines $y(a,z)$. If it defaults, it stays at the state of default forever. If it repays, it chooses how much to borrow from risk neutral investors B' and investors purchase government bonds. In equilibrium, $B' = b'$. This determines g.

A comment is in order concerning the difference between our timing assumption and the assumption by the literature after Eaton and Gersovitz (1981) and Cole and Kehoe (2000). In this literature, the government sets the total issuance of bonds

[6] Corsetti and Dedola (2016), Ayres et al. (2018) and Lorenzoni and Werning (2019) adopt the same timing assumption.

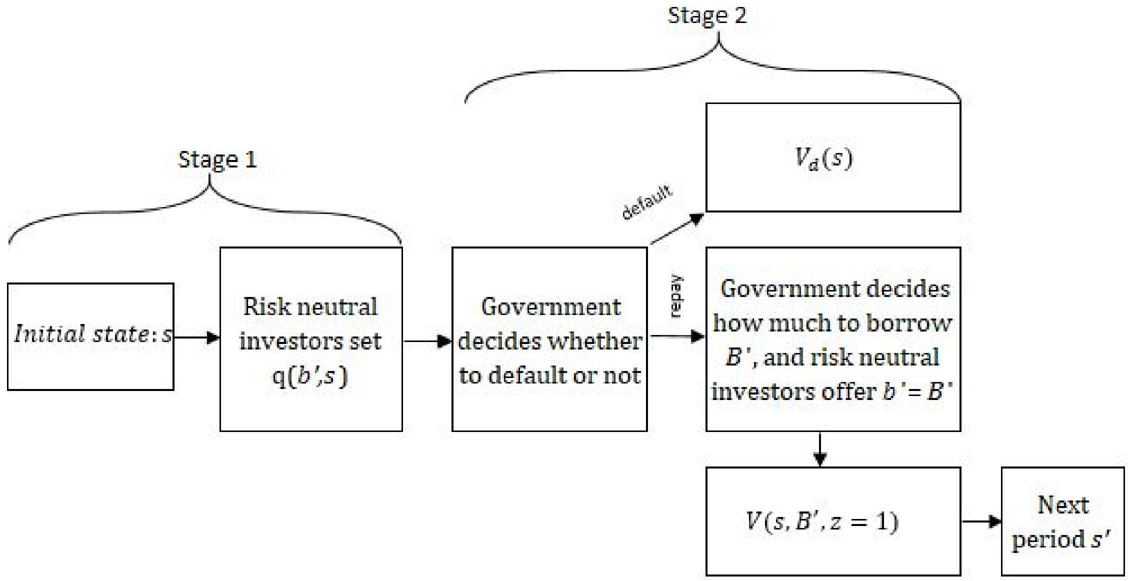

Figure 1: Timing description

at face value; investors set bond prices afterwards. In Cole and Kehoe (2000), this timing assumption acts as a selection criterion that rules out crises of the type analyzed by Calvo (1988).[7] Implicit in this assumption, however, is that the government commits to issue a predetermined amount of bonds, and is able to adjust its primary surplus to make up for any shortfall in fiscal revenue if the equilibrium bond price turns out to be lower than anticipated. The point of departure of our analysis is to relax this assumption, and allow the government to adjust both its fiscal surplus and its bond issuance to market conditions—that is, vis-à-vis possible multiple values for the equilibrium bond prices $q(b', s)$ at stage 1 in Figure 1. In line with the arguments by Lorenzoni and Werning (2019), we find this timing assumption a plausible working hypothesis.

2.1 Bond pricing

For tractability, we follow the literature and assume that investors are risk neutral and discount the future using the factor β. The bond prices $q(b', s)$ are therefore determined by the probability that investors assign to default in the future. Denoting by x (the linear) consumption, the investors' problem is:

[7] In this model, provided investors are willing to finance the government, they offer the best equilibrium price for the bonds.

$$
\begin{aligned}
W(b,s) = \max_{x,b'} \quad & x + \beta \mathbb{E}[W(b',s')] \\
\text{s.t.} \quad & x + q(b',s)b' = w + z(B'(q(b',s),s),s,q(b',s)) \times \\
& \qquad (\kappa b + q(b',s)(b' - (1-\delta)b)), \\
& \qquad\qquad x \geq 0, b \geq -\mathcal{A}
\end{aligned}
$$

The constraint $b \geq -\mathcal{A}$ imposes the no-Ponzi condition, but $\mathcal{A}$ is set large enough that the constraint never binds. By the same token, x, the linear consumption of investors, is set large enough (deep pockets assumption) to rule out corner solutions.

As in Conesa and Kehoe (2017), there are two cutoff levels of debt each period, $\bar{B}(a)$ where $a = 0, 1$, with $\bar{B}(0) \leq \bar{B}(1)$:

1. If $B \leq \bar{B}(0)$, the government does not default, independently of the business cycle.
2. If $B \leq \bar{B}(1)$, the government does not default provided the economy is not in a recession.

For future reference, we find it insightful to dub these thresholds as the "debt tolerance" of the country.

Recall that, once the government defaults ($z = 0$), z stays at 0 forever. This assumption implies that the equilibrium bond price is zero in any history with past default:

$$q(b', (B, 0, a)) = 0$$

If the government had not defaulted in previous periods, the first order condition of risk neutral investors' problem implies:

$$
\begin{aligned}
q(b',s) = \beta \mathbb{E}\Big[z\Big(B'(q(b'(s'),s'),s'), s', q(b'(s'),s')\Big)(\kappa + (1-\delta) q(b'(s'),s'))\Big] \\
\text{where } s' = s'\big(q(b'(s),s)\big)
\end{aligned} \tag{1}
$$

It is well understood that the source of equilibrium multiplicity in the model is rooted in the possibility that multiple values of $q(b', s)$ solve (1).

In equilibrium, $q(b', s)$ is consistent with market clearing condition $b' = B'$ which implies that one possible bond price function can be

$$q(B', (B, 1, 0)) = \begin{cases} \beta\big[\kappa + (1-\delta)\mathbb{E}[q'(\cdot)]\big] & \text{if } 0 \leq B' \leq \bar{B}(0) \\ \beta p & \text{if } \bar{B}(0) < B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

in a recession and

$$q(B', (B, 1, 1)) = \begin{cases} 1 & \text{if } 0 \leq B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

in normal times.

2.2 Government optimization problem

Given the bond price function $q(b', s)$, if the government decides not to default, it chooses its fiscal deficit and issues $b' = B'$ at the equilibrium bond price $q(B', s)$. The government's problem can be reduced to choose B', z to solve

$$\begin{aligned} V(s) = \max \quad & u(c, g) + \beta\mathbb{E}[V(s')] \\ \text{s.t.} \quad & g + z\kappa = \theta y(a, z) + zq(B', s)(B' - (1-\delta)B), \\ & c = (1-\theta)y(a, z), \\ & z = 0 \text{ if } z_{-1} = 0 \end{aligned} \tag{2}$$

As in Conesa and Kehoe (2017), we posit that, for any B, the following condition holds

$$u_g((1-\theta)A\bar{y}, \theta A\bar{y} - \kappa B) > u_g((1-\theta)\bar{y}, \theta\bar{y} - \kappa B)$$

This ensures that, in a recession, the government always has an incentive to raise debt due to the high marginal benefit of government spending when the economy is in a bad state.

In the framework presented above, the government defaults if and only if the utility of repaying debt V_n is smaller than the utility of defaulting V_d:

$$V_n < V_d$$

The value of defaulting is determined by assuming that, in case of debt repudiation, the country loses market access *and* experiences a discrete contraction in output by Z—output stays at $AZ\bar{y}$ in a recession and $Z\bar{y}$ in normal times. For tractability, both costs are assumed to be permanent. This will define our baseline default model.

2.3 Equilibrium

An equilibrium is a value function for the government $V(s,q)$ and policy functions $B'(s,q)$, $z(s,q)$ and $g(s,q)$, a value function for investors $W(b,s)$, policy function $b'(b,s)$, and an equilibrium bond price function $q(B',s)$ such that

1. Given policy function $z(s,q)$, $g(s,q)$, $V(s,q)$ and $B'(s,q)$, $b'(b,s)$ solves investors' problem at the beginning of the period and $q(B',s)$ is consistent with market clearing and rational expectations.

2. $V(s,q)$, $B'(s,q)$, $z(s,q)$ and $g(s,q)$ solve government's optimization problem in (2) given bond price function $q(B',s)$.

For tractability, the notion of equilibrium we consider follows a simple Markov structure.

2.4 Debt limit

Relative to the baseline model specified above, one may envision economies in which the government is averse to default, yet the prevailing conditions in the economy and the market may undermine its ability to honour its liabilities. In Ghosh et al. (2013) and Lorenzoni and Werning (2019), default is decided against a given path of maximum (contingent) primary surpluses that the government can generate. Default occurs if and only if the amount the government can borrow from the market is not enough to finance its interest bill, given this path. The condition for the government to default in a recession would be:

$$\max_{B'}\{q(B',s)(B'-(1-\delta)B)\} < \kappa B - \underbrace{(\theta A\bar{y} - \bar{g})}_{\text{max primary surplus}} \tag{3}$$

If the stock of initial debt is high, by borrowing, the government can smooth the adjustment in spending and taxation required to service debt across many periods. But given the path of maximum primary surpluses, bond prices may move against the government to such an extent that the required adjustment in the short run become intolerable. We will show how to nest this model in our framework and analyze the implications in section 6.

3 Equilibrium multiplicity with short-term debt

In this section, we specialize the baseline model laid out in the previous section, assuming that debt is short-term only. The economy can be plagued by multiple equilibria: debt crises can be driven by a switch in the regime of investors' expectations across these equilibria. Debt crises can be either "slow-moving" or "fast" (rollover). Intuitively, slow-moving crises occur for intermediate levels of debt, where a switch in expectations causes borrowing costs to rise, but the government can deliver enough adjustment for the market to keep satisfying its financing need at the higher equilibrium rates. Fast (rollover) crises occur at higher level of debt, because once investors turn pessimistic, the government may not be willing (or able as in section 6) to contain its deficit enough to obtain financing at any finite equilibrium interest rates. Knowing this, investors simply refuse to rollover the government debt. If and when investors become pessimistic, they "run" and the government loses market access.

As discussed below, key to these results is that the government debt tolerance, i.e., the debt default thresholds, may change with investors' expectations. If investors become "pessimistic" about debt sustainability, so that in equilibrium the cost of borrowing for the government rises, the government may default at lower level of debt, relative to equilibrium where investors are "optimistic". The size of the shift relative to the initial stock of debt determines whether the crisis is slow-moving or fast. We should emphasize once more that the "rollover crises" in our model develop in a different institutional framework compared to the seminal model by Cole and Kehoe (2000).

We start by analyzing the debt tolerance thresholds in an expansion and in a recession. Then we study and interpret the equilibrium.

3.1 Debt tolerance thresholds in normal times and recessions

In our notation, debt tolerance thresholds are contingent on the state of the economy—we write $\bar{B}(a)$ $a = 0, 1$: $\bar{B}(0)$ is the maximum sustainable debt level in a recession and $\bar{B}(1)$ is the maximum sustainable debt level in normal times. Let $V_n(B, z_{-1}, a)$ and $V_d(a)$ denote, respectively, the government's utility if the government repays its debt, and the government's utility of defaulting, assessed either in normal times $(a = 1)$ or in a recession $(a = 0)$.

3.1.1 The debt tolerance threshold in normal times, $\bar{B}(1)$.

The derivation of $\bar{B}(1)$ is straightforward, since under our simplified assumption the government's optimization problem is deterministic after the economy recovers (a stays at 1 forever). In this case, without loss of generality, we can abstract from multiplicity. If the government decides to repay existing debt, it pays $(1-\beta)B$ each period to obey no-Ponzi condition. The government utility conditional on repaying its debt is

$$V_n(B, 1, 1) = \frac{u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B)}{1-\beta}$$

Write the utility of defaulting when the economy is not in a recession as

$$V_d(1) = \frac{u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta}$$

It follows that $\bar{B}(1)$ can be characterized by solving:

$$V_n(\bar{B}(1), 1, 1) = V_d(1)$$

The debt tolerance threshold $\bar{B}(1)$ is unique in that investors hold a unique consistent view, that the economy will remain in normal times forever (there is no output uncertainty any more). Fundamental risk will instead be crucial in generating multiple debt tolerance thresholds when the economy is in a recession.

3.1.2 The debt tolerance threshold(s) in a recession $\bar{B}(0)$.

To determine the threshold in a recession, it is useful to distinguish from the start the possibility that investors can hold consistent beliefs that are either "pessimistic" or "optimistic".

When investors are pessimistic, given the equilibrium financing need of the government, they attribute probability one to a default in the following period if a recession persists. In this case, investors offer a low bond price βp to the government. While the utility of defaulting in a recession is independent of expectations:

$$V_d(0) = \frac{u((1-\theta)AZ\bar{y}, \theta AZ\bar{y})}{1-\beta(1-p)} + \beta\frac{pu((1-\theta)Z\bar{y}, \theta Z\bar{y})}{(1-\beta)(1-\beta+\beta p)}$$

the utility of repaying debt given the low bond price is not. Denoting this utility with the subscript "pes", to stress that this may depend on expectations, we have:

$$\begin{aligned} V_{pes}(B,1,0) = \max_{0\le B'\le \bar{B}(1)} \quad & u(c,g) + \beta\Big(\frac{p}{1-\beta}u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B') \\ & \qquad + (1-p)V_d(0)\Big) \\ \text{s.t.} \quad & g + B = \theta A\bar{y} + \beta p B', \\ & c = (1-\theta)A\bar{y} \end{aligned} \tag{4}$$

To determine the debt threshold $\bar{B}(0)_{pes}$, we solve the equation below:

$$V_{pes}(\bar{B}(0)_{pes}, 1, 0) = V_d(0)$$

To ensure $\bar{B}(0)_{pes}$ is self-fulfilling, the choice variable $B'(\bar{B}(0)_{pes})$ in the value function (4) must be larger than $\bar{B}(0)_{pes}$.

Conversely, when investors are optimistic, they presume that the government will be willing and able to service its debt even if the economy remains in a recession in the next period. Hence, in writing the utility of repaying debt, the equilibrium bond price is riskless: $V_{opt}(B,1,0) = \max\{V_{opt,1}(B,1,0), V_{opt,2}(B,1,0)\}$. Note that we allow for the possibility that the debt issuance capacity of the government is constrained by a debt threshold below the one conditional on the economic recovery.

In particular, either of the following may be relevant:

$$V_{opt,1}(B,1,0) = \max_{0 \leq B' \leq \bar{B}(0)_{opt}} \quad u(c,g) + \beta\Big(\frac{p}{1-\beta}u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B') + (1-p)V_{opt}(B',1,0)\Big)$$

$$\text{s.t.} \quad g + B = \theta A\bar{y} + \beta B',$$

$$c = (1-\theta)A\bar{y}$$

$$V_{opt,2}(B,1,0) = \max_{\bar{B}(0)_{opt} < B' \leq \bar{B}(1)} \quad u(c,g) + \beta\Big(\frac{p}{1-\beta}u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B') + (1-p)V_d(0))\Big)$$

$$\text{s.t.} \quad g + B = \theta A\bar{y} + \beta p B',$$

$$c = (1-\theta)A\bar{y}$$

The debt threshold $\bar{B}(0)_{opt}$ is again the solution of the equation below.[8]

$$V_{opt}(\bar{B}(0)_{opt}, 1, 0) = V_d(0)$$

To gain insight on the thresholds characterized above, in the next subsection we rely on a simple graphical apparatus.

3.2 An intuitive graphical analysis

Figure 2 includes three panels, each depicting both the optimistic and the pessimistic regime of expectations together. In each panel, the x axis measures the amount of bond the government issues during the period, the y axis measures the resources that the government can obtain by issuing debt at the equilibrium price, qB'. Vertical dashed lines denote the debt tolerance thresholds derived above. For our graphical analysis, we find it convenient to introduce a new debt threshold, labelled B_N, which denotes the maximum amount of the *initial* debt level in a recession below which the country is immune to pessimism—as shown below, below B_N, investors will always purchase debt at the risk free price.

[8]The algorithm for computing an equilibrium in a recession is shown in online Appendix A.

In each panel, a solid blue line depicts the Laffer Curve conditional on optimistic expectations, a red line the Laffer Curve conditional on pessimistic expectations. Note that both curves have a discontinuity. Focus first on the case of optimistic expectations (blue line). From the origin to the threshold $\bar{B}(0)$, the Laffer curve has slope β, the risk-free bond price—as long as new issuance does not increase the stock of debt above the threshold $\bar{B}(0)$ government debt is free of default risk. Beyond this threshold, the price of debt falls as investors anticipate contingent default one period ahead. Due to default risk, any issuance bringing the debt stock in the range between $\bar{B}(0)$ and $\bar{B}(1)$ will be priced βp: the Laffer Curve has a flatter slope. Similar considerations extend to the Laffer curve conditional on pessimistic expectations (red line)—except that the discontinuity now occurs at B_N.

In each panel, also, the downward sloping line draws the endogenous Gross Financing Need of the government, as a function of the price of bonds and the initial debt. The GFN line is downward sloping because at lower bond prices (higher interest rates) the government optimally reduces its primary deficit. Intuitively, moving down the GFN line, think of each point as crossing a ray from the origin (not shown), corresponding to a lower bond price q. At lower q's, the government faces higher borrowing costs. The government has thus an incentive to adjust its spending optimally, reducing its financing need. However, this optimal adjustment falls short of reducing the new issuance of debt, hence the GFN line is decreasing monotonically. The initial level of debt instead determines the position of the GFN in the graph. A higher stock of liabilities inherited by the government moves the GFN out to the right.

It is worth stressing that, in the model and in the graph, the Laffer curves and the debt tolerance thresholds differ across the optimistic and the pessimistic scenarios, but are independent of the initial debt level. Hence the pair of Laffer curves (one for the optimistic, the other for the pessimistic scenario) are exactly the same across the three panels of Figure 2.

The left panel of Figure 2 illustrates a case in which the initial debt stock is so low that the equilibrium is unique and bonds are traded at default-free prices. In the panel, the GFN intersects the debt Laffer curve in two points, at L_{opt} and L_{pes}. In the first point, debt is issued at risk-free rate; in the second point, debt

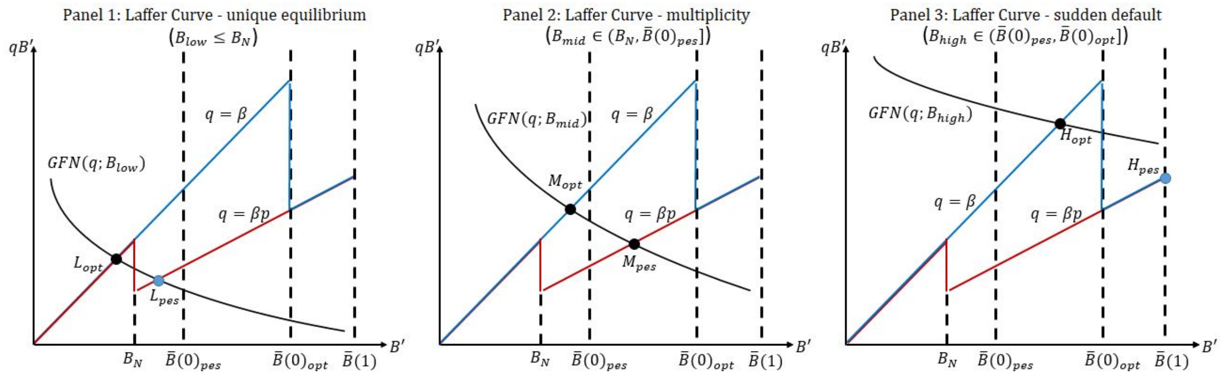

Figure 2: Multiple equilibria and unique equilibrium with $\delta = 1$

is default-risky. It is easy to verify that the latter cannot be an equilibrium. Even if investors become pessimistic, the government would still issue debt below the tolerance threshold $\bar{B}(0)_{pes}$, since its GFN is moderate. Investors' pessimistic view would not be validated ex-post. The only self-fulfilling equilibrium is at the point L_{opt}, with riskless pricing in equilibrium. In other words, in panel 1, $q = \beta p$ is not the solution to investors' first order condition (1), shown below:

$$\underbrace{q(b', s)}_{=\beta p} \neq \beta \mathbb{E}\left[\overbrace{z\Big(\underbrace{B'(q(b'(s'), s'), s')}_{<\bar{B}(0)}, s', \underbrace{q(b'(s'), s')}_{\bar{B}(0)=\bar{B}(0)_{pes}} \Big)}^{=1} \right] \tag{5}$$

Using this equation, we can determine at which level of debt the equilibrium is no longer unique, B_N. This threshold can be found by solving the equation (6) below:

$$B_N = \sup_{b'} \left\{ \underbrace{q(b', s)}_{=\beta p} \neq \beta \mathbb{E}\left[\overbrace{z\Big(\underbrace{B'(q(b'(s'), s'), s')}_{<\bar{B}(0)}, s', \underbrace{q(b'(s'), s')}_{\bar{B}(0)=\bar{B}(0)_{pes}} \Big)}^{=1} \right] \right\} \tag{6}$$

For *initial debt levels* larger than B_N, if investors develop a pessimistic view on government solvency, an equilibrium with default can be self-validating: the government either borrows more than $\bar{B}(0)_{pes}$, or default immediately. We now turn to these cases, looking at panels 2 and 3.

In the center panel of the figure, the government initial debt B_{mid} is at intermediate level. Precisely, B_{mid} is larger than B_N but not too high—smaller than $\bar{B}(0)_{pes}$. Similar to the left panel, the GFN function intersects the debt Laffer curves in two points, M_{opt} and point M_{pes}. Both points can now be an equilibrium. When investors buy newly issued sovereign debt at the riskless price β, overall borrowing will remain below the relevant debt tolerance threshold, $\bar{B}(0)_{opt}$, validating ex-post the investors' optimistic view. By contrast, new issuance will go beyond (lower) tolerance threshold $\bar{B}(0)_{pes}$ when investors buy at the lower risky price. One period ahead, unless the economy recovers, the government defaults, validating investors' pessimism. For both $q = \beta$ and $q = \beta p$ to be equilibrium prices, the initial stock of debt must be such that the equations (7) and (8) from the investors' first order condition (1) are satisfied at once:

$$\underbrace{q(b', s)}_{=\beta} = \beta \mathbb{E}\left[\overbrace{z\Big(\underbrace{B'(q(b'(s'), s'), s')}_{<\bar{B}(0)}, s', \underbrace{q(b'(s'), s')}_{\bar{B}(0)=\bar{B}(0)_{opt}} \Big)}^{=1} \right] \tag{7}$$

$$\underbrace{q(b', s)}_{=\beta p} = \beta \mathbb{E}\left[\overbrace{z\Big(\underbrace{B'(q(b'(s'), s'), s')}_{>\bar{B}(0)}, s', \underbrace{q(b'(s'), s')}_{\bar{B}(0)=\bar{B}(0)_{pes}} \Big)}^{=p} \right] \tag{8}$$

The type of equilibrium with belief-driven default shown in the center panel of Figure 2 corresponds to a scenario in which, as stressed by Lorenzoni and Werning (2019), the debt crisis is 'slow-moving'. Interest rates are high because investors expect the government to default if a recession persists. Because of high borrowing costs, the stock of government debt rises prior to default. But default only occurs if and only if the country remains in a recession in the future.

The right panel of Figure 2 shows that the model admits another type of belief-driven default in equilibrium, possible for a relatively high initial debt level, higher than $\bar{B}(0)_{pes}$. The GFN line now intersects the Laffer curves at the point H_{opt}. If investors buy government bonds at the riskless price β, despite the high stock of initial liabilities, new debt issuance remains below the relevant threshold, $\bar{B}(0)_{opt}$. However, if investors turn pessimistic, the hike in borrowing rates causes the government to become much less 'tolerant' of the adjustment required to service the debt. At the point H_{pes}, investors anticipate that the government will not be able and/or willing to adjust its primary needs enough to keep new issuance of debt below $\bar{B}(1)$ at the default-risk bond price, and thus a "fast" debt crisis occurs.

We can look at this "fast" debt crisis from two angles. From the vantage point of the government, when the market expects a default in a recession, new bonds can only be issued at risky rates. But at these rates, even after adjusting its primary surplus, the government is unable to satisfy its financing need keeping debt issuance below its maximum debt capacity in normal times—beyond which a default occurs for sure. Even if investors were willing to charge finite interest rates (presumably conditional on the government cutting its deficit further), immediate default would be the preferred option. Anticipating all this, from the investors' vantage point, it is rational not to finance the government at all: the country instantly loses market access. In a rollover crisis, the government has no alternative but to default. The condition is given by equation (9) below.

$$\underbrace{q(b', s)}_{=0} \;=\; \beta \mathbb{E}\left[\overbrace{z\Big(B'(q(b'(s'), s'), s'), s', \underbrace{q(b'(s'), s')}_{\bar{B}(0)=\bar{B}(0)_{pes}}\Big)}^{=0} \right] \tag{9}$$

It is important to clarify why the point H_{pes} in the figure is not an equilibrium. This is shown in equation (10). Once investors turn pessimistic, the government optimally cuts its deficit and reducing its current financing need, moving down along the GFN line. In principle, the government could implement further cuts in its deficit, but this would never be optimal given that spending and utility remains relative high after default, i.e., given that post default output remains sufficiently high relative to the critical expenditure level $\bar{g}$.

$$\underbrace{q(b', s)}_{=\beta p} \;\neq\; \beta \mathbb{E}\left[\overbrace{z\Big(B'(q(b'(s'), s'), s'), s', \underbrace{q(b'(s'), s')}_{\bar{B}(0)=\bar{B}(0)_{pes}}\Big)}^{=0} \right] \tag{10}$$

The right panel in Figure 2 thus features the possibility of debt crises that we dub "fast". There is no "slow-moving" increase in debt levels, leading up to a crisis. Upon the drop of the debt tolerance thresholds in response to changes in investors' views on government solvency, a debt crisis arrives "fast" and "early".

4 Sustainability and crises with long-term debt

For any given stock of debt, longer maturities may help sustainability, by reducing the exposure to rollover risk and the pass-through of hikes on interest rates onto the total cost of servicing the outstanding debt. An important question is whether and under what circumstances maturity can rule out multiplicity leading to either slow-moving or to fast debt crises.

In this section, we show that multiplicity of equilibria remains pervasive. To do so, we rely on a numerical example, calibrating our model with standard parameter values.

4.1 Calibration

In solving the model with long-term debt, we posit the following functional forms for the utility function:

$$u(c, g) = \log(c) + \gamma \log(g - \bar{g}); \tag{11}$$

In our calibration, we set benchmark parameters following Conesa and Kehoe (2017). The parameter values are shown in Table 1.

Table 1: Parameter values, baseline

$\bar{y}$	Output	100
Z	Cost of Default	0.95
β	Discount factor	0.98
γ	Relative weight of c and g in the utility function	0.20
θ	Government revenue as a share of output	0.36
$\bar{g}$	Level of the critial government expenditure	25
A	Fraction of output during recession	0.9
p	Probability of leaving the recession	0.2
δ	Amortization rate of market debt	0.2

As shown in the table, we normalize output $\bar{y}$ to 100 so that the units in the model can be interpreted as percentage of GDP: e.g. $B = 50$ means that debt to GDP ratio is 50% in normal times. We set cost of default as $5\% = 1 - Z$, and this cost of default is permanent. Our default cost is lower relative to the literature (e.g.

Alesina et al. (1992)), on the grounds that we assume this cost to be permanent.[9] We assume the relative weight of government utility is 0.2; sensitivity analysis shows that this parameter is unimportant for our result.

The severity of recession A is set to 0.9 so that a recession results in a decrease in output by 10% for the benchmark scenario. This parameter is crucial to generating gambling for recovery in an optimistic world. A more severe recession leads to a stronger smoothing motive for the government, which may induce the government to choose a high-debt risky-debt strategy—we report results for different A in sensitivity analysis.

We set the critical government expenditure $\bar{g}$ at 25% of GDP: the higher this value, the smaller the room for discretionary spending. Government revenue as a fraction of output is determined by the constant tax rate θ. In normal times, the government's income is 36, but in a recession, it drops to 32. We posit $\delta = 0.2$ to match average maturity from 2000-2009 for Greece, Italy and Spain, which is about 5 years. We set $p = 0.2$ so that the expected waiting time for recovery is 5 years.

The key novel result from our analysis is that, since the debt tolerance threshold in a recession moves with investors' expectations, this might result in "fast" debt crises—a result that holds also when debt has long maturity. Figure 3 plots the policy functions conditional on a recession, together with the debt tolerance thresholds (in a recession and in normal times), in the optimistic world (left panel) and the pessimistic world (the right panel).

A striking feature of the optimistic world—on the left panel of Figure 3—is the high value of the debt tolerance threshold in a recession, about 204% of GDP (or 184% as a ratio of GDP in normal times). Notably, in our exercises, we find that $\bar{B}(0)_{opt}$ is not sensitive to the probability of recovery p or debt maturity δ, but quite sensitive to A, as further discussed in section 7. In a recession, the government will smooth consumption by borrowing at risk-free rate utill debt level reaches $\bar{B}(0)_{opt}$: the figure suggests that the dynamic of debt is mildly increasing.

The right panel of Figure 3 depicts a situation in which investors unexpectedly change their view on government solvency, from optimistic to pessimistic. While $\bar{B}(1)$ is not affected, by virtue of our assumption that, after recovering, the economy

[9] Upon a default, in our baseline scenario Z cannot be too small in that the government spending cannot fall below $\bar{g}$. In other words, the conditions $\theta AZ\bar{y} > \bar{g}$ and $\theta Z\bar{y} > \bar{g}$ must be satisfied.

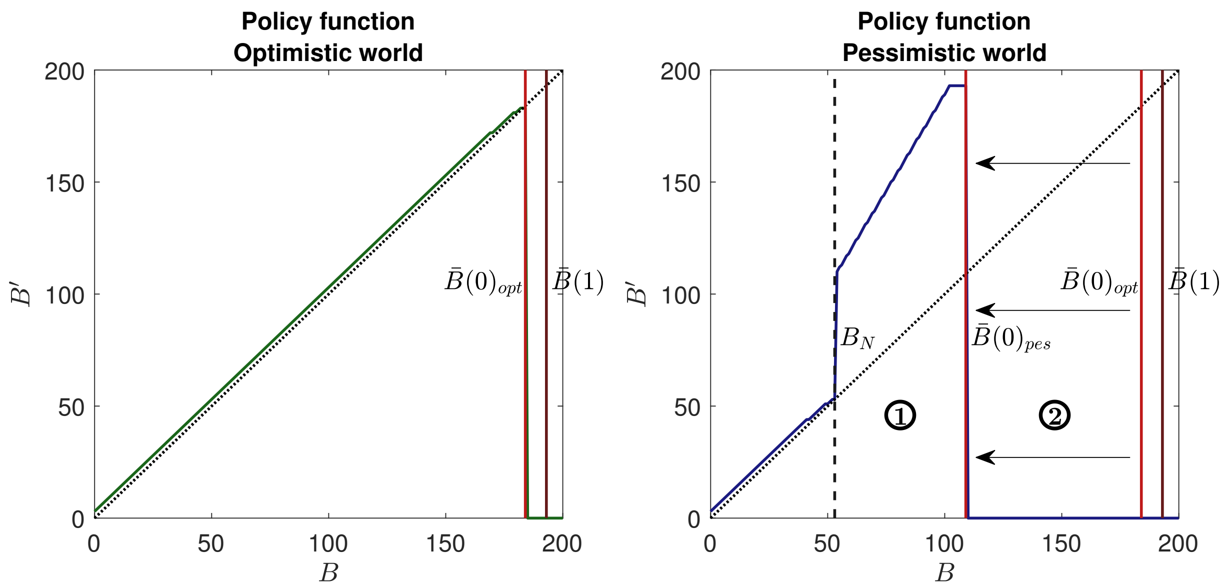

Figure 3: Policy functions in the optimistic world and the pessimistic world

never falls back into a recession again, the consequences of such a change on the debt tolerance threshold in a recession are stark. There is a large drop from $\bar{B}(0)_{opt}$ to $\bar{B}(0)_{pes}$.

If the initial debt is in the region between 0 and B_N, the country is barely affected by the switch in expectations. The government is still able to borrow at risk-free rate, and, as a result, it keeps increasing the level of debt for smoothing purposes till the stock of debt reaches B_N, and then remains there, waiting for a recovery.

If the initial debt is anywhere above B_N but below $\bar{B}(0)_{pes}$ in a pessimistic world, the government will pay high rates of interest and its debt will start to accumulate at a faster pace. In the region labelled ① in the figure, default may then occur depending on whether the economy fails to recover in the next period. This is the scenario of a "slow-moving" crisis: a default is preceded by debt accumulation. Note that, under our parameterization, a slow-moving crisis can arise for a debt to GDP ratio as low as 53% of GDP in normal times (about 59% of GDP in a recession).

If debt is in the region between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$—the region labelled ② in the figure—the crisis will be of the type that we dub "fast": it will occur in response to the shift in expectations. Observe that in the "fast crisis" region, as long as investors are optimistic, the government can actually issue debt at risk-free rate. But once investors change their view and charge high risky rates, they understand that the sovereign will be unwilling to reduce its financing need enough to keep new issuance below $\bar{B}(1)$. The debt market dries out. Facing such a rollover crisis, the government defaults immediately. There is no "slow-moving" accumulation of debt. In our calibration, fast crises can occur with a debt to GDP ratio in a recession between about 121% and 204%.

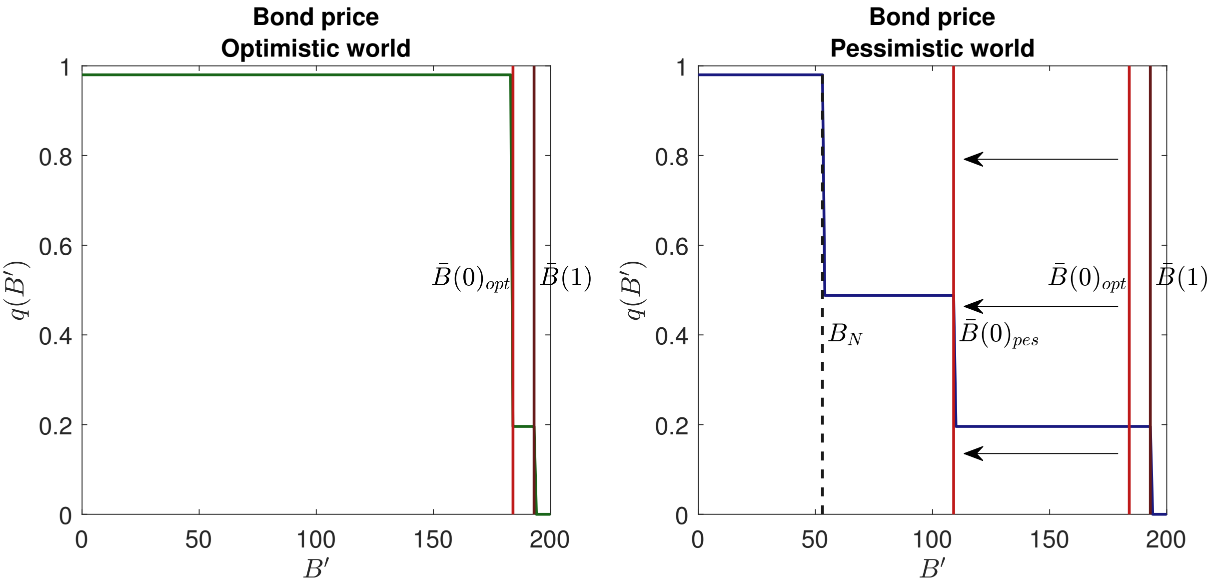

Figure 4: Bond prices in the optimistic and the pessimistic world

In Figure 4, we plot the price of government bonds in both the optimistic and the pessimistic worlds, contingent on a recession. The left panel in the figure shows that this price remains high for a wide range of debt-to-GDP ratio in the optimistic world. The price drops very markedly in the narrow region between $\bar{B}(0)_{opt}$ and $\bar{B}(1)$. The right panel of Figure 4 illustrates the impact of a change in investors' expectations, from optimistic to pessimistic. If the amount of new issuances lies between B_N and $\bar{B}(0)_{pes}$, the government may be exposed to the probability of slow-moving debt crises next period. Thus, the bond price drops to 0.48. If it issues above $\bar{B}(0)_{pes}$ but below $\bar{B}(1)$, the "fast" debt crises may occur, which decreases the bond price even further.

To highlight the role of debt maturity, Figure 5 shows the policy functions conditional on a recession for the case of one-period bonds ($\delta = 1.0$). Comparing this with Figure 3, it is apparent that, as δ converges to unity, $\bar{B}(0)_{pes}$ is much lower, while $\bar{B}(0)_{opt}$ is not affected. The result that $\bar{B}(0)_{opt}$ is not sensitive to debt maturity follows from the fact that, when investors hold an optimistic view on government solvency, they lend the government at risk-free rate: thus there is little scope for

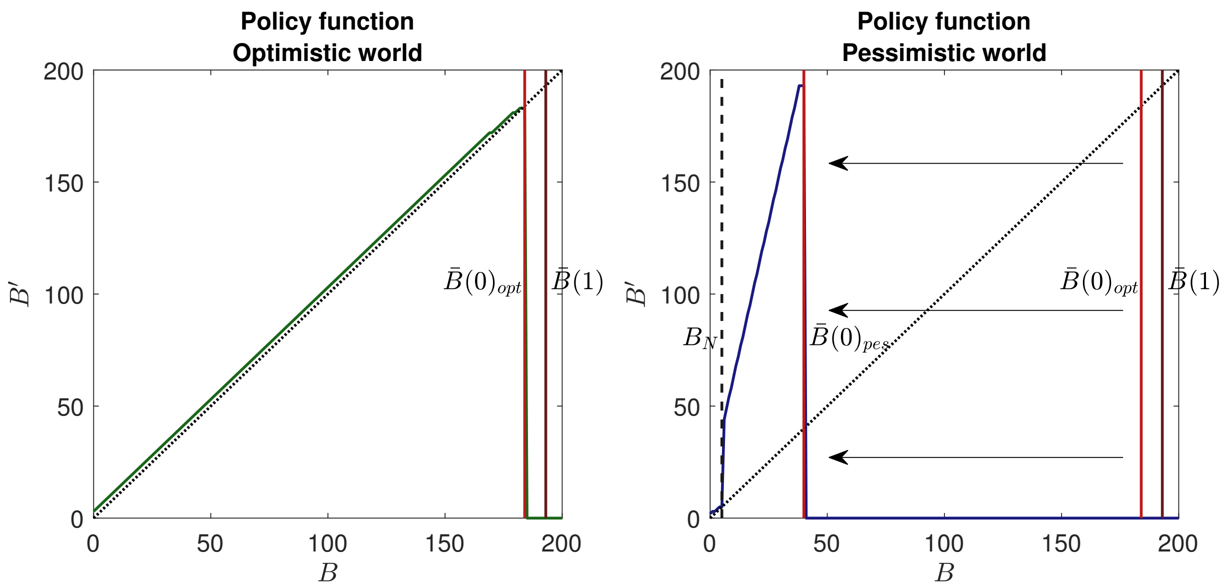

Figure 5: Policy functions for one-period bonds, $\delta = 1.0$

maturity to make a difference. Indeed, the left panel in Figure 5 features exactly the same dynamics as the left panel of Figure 3.

Maturity instead makes a difference for the region of debt in which "fast" and "slow crises". The region between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$ is much wider with short-term debt. The rollover crises might occur for low level of debt (above 40% of GDP in normal times). Moreover, a larger region of "fast crises" is not fully compensated by a narrowing of the region of "slow crises", as both B_N and $\bar{B}(0)_{pes}$ shrink when maturity is shorter. B_N falls from 53 to 5! When government bonds are all short-term, a country in a recession might suffer a "slow-moving" crisis even if it has negligible outstanding debt.

Summing up: a long debt maturity can substantially improve government's welfare by increasing debt thresholds in a pessimistic world, but it may not rule out any types of self-fulfilling crises. Threats of both "slow-moving" and "fast" debt crises are still pervasive with long-term debt. In section 6, we will show that the same does not necessarily apply to the debt-limit framework.

5 Does the threat of self-fulfilling crises motivate debt deleveraging?

So far we have carried out our analysis under the implicit assumption that, when in an optimistic mode, investors and the government attribute zero probability to the bad equilibrium. In this section, we relax this assumption and construct sunspot equilibria, heavily drawing on the approach by Conesa and Kehoe (2017). Namely, we posit that investors are initially optimistic on government solvency, but all agents in the economy are aware that market views may turn pessimistic with probability π. If this pessimistic view is self-fulfilling (in that the government borrows more than $\bar{B}(0)_{pes}$ once the switch occurs), investors remain pessimistic forever afterwards. Debt tolerance threshold in a recession will now be denoted as $\bar{B}(0)_{\pi}$. We posit a small sunspot probability, equal to $\pi = 0.04$.

Our key result is that, in a sunspot equilibrium, the government may choose to decrease debt to safe levels in a recession, motivated by large gains in expected utility from either eliminating sunspot crises altogether (we dub this the welfare 'cliff effect' of belief-driven crises), or lowering borrowing costs (the 'price effect'), or both. However, different from Cole and Kehoe (2000), deleveraging will be preferred over debt accumulation only for a small range of debt above the threshold at which slow-moving crises become a possibility. For a very wide range of debt levels, the government prefers to accumulate liabilities and smooth consumption, gambling on the prospective recovery.

Figure 6 displays the policy function (left) and the bond price function (right) in sunspot equilibrium with long-term debt. For debt levels in the region between 0 and B_N, the debt dynamics are the same as in the right panel of Figure 3, and the government is able to issue safe debt.

In the region between B_N and $\bar{B}(0)_{\pi}$, where the economy is vulnerable to sunspot crises, the debt dynamics are different from what we have seen so far—it is no longer uniform. This region can indeed be split into two subregions. For an initial debt level close to B_N, the government chooses to run surpluses and reduce its borrowing. This allows the government to avoid high and increasing borrowing costs, as well as a large utility loss if self-fulfilling pessimistic expectations materialize. However, for

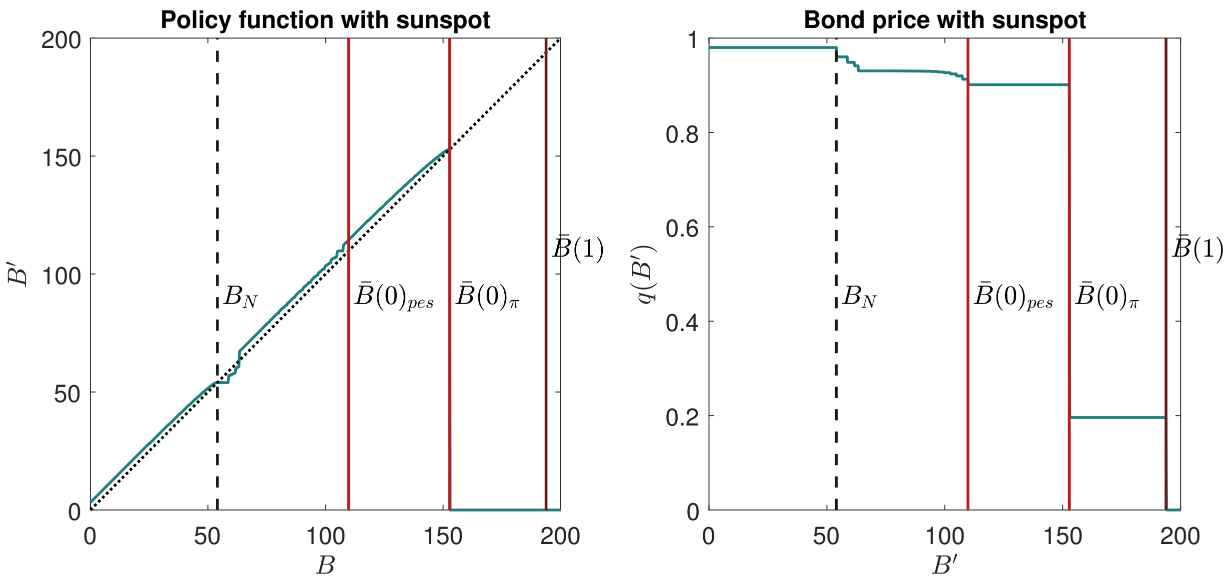

Figure 6: $\delta = 0.2$, $A = 0.9$, $p = 0.2$ with sunspot

a larger initial debt, the government prefers to keep borrowing. It will do so until its debt level reaches $\bar{B}(0)_{\pi}$, even for debt levels above (but close to) $\bar{B}(0)_{pes}$, where self-fulfilling crises, if they occur, are "fast".

Why is deleveraging optimal for debt levels just above B_N, but not so for debt levels just above $\bar{B}(0)_{pes}$? The key insight is that keeping debt below $\bar{B}(0)_{pes}$ shields the country from "fast" crises, but not from slow-moving ones. Hence, while the government may still have some advantage not to let debt trespass $\bar{B}(0)_{pes}$, this advantage is exclusively in terms of lower borrowing costs (as shown on the right panel of Figure 6), not in terms of eliminating the possibility of crises 'tout-court' (the 'cliff effect' is less relevant here).[10] The borrowing costs advantage (about 1.4%)[11] is not enough to offset the need to smooth consumption in a recession via borrowing. The government exposed to the risk of fast crises de facto accumulates debt faster than slow moving one.[12].

Things are different when government debt is short-term, as the 'price' and 'cliff effects' play a somewhat different role in shaping government decisions. The case of one-period bonds, with $\delta = 1.0$, is shown in Figure 7. Besides the fact that levels

[10]Discontinuity in value function, like a 'cliff' in a pessimistic world, motivates the government to deleverage. See online Appendix B for details.

[11]This is obtained by $\big(1 - q(\bar{B}(0)_{pes} + 1)\big)/q(\bar{B}(0)_{pes} + 1) - \big(1 - q(\bar{B}(0)_{pes})\big)/q(\bar{B}(0)_{pes})$. We use the same formula to derive yield difference of one-period bonds in our next simulation, which is 3.4%.

[12]This is shown in online Appendix C

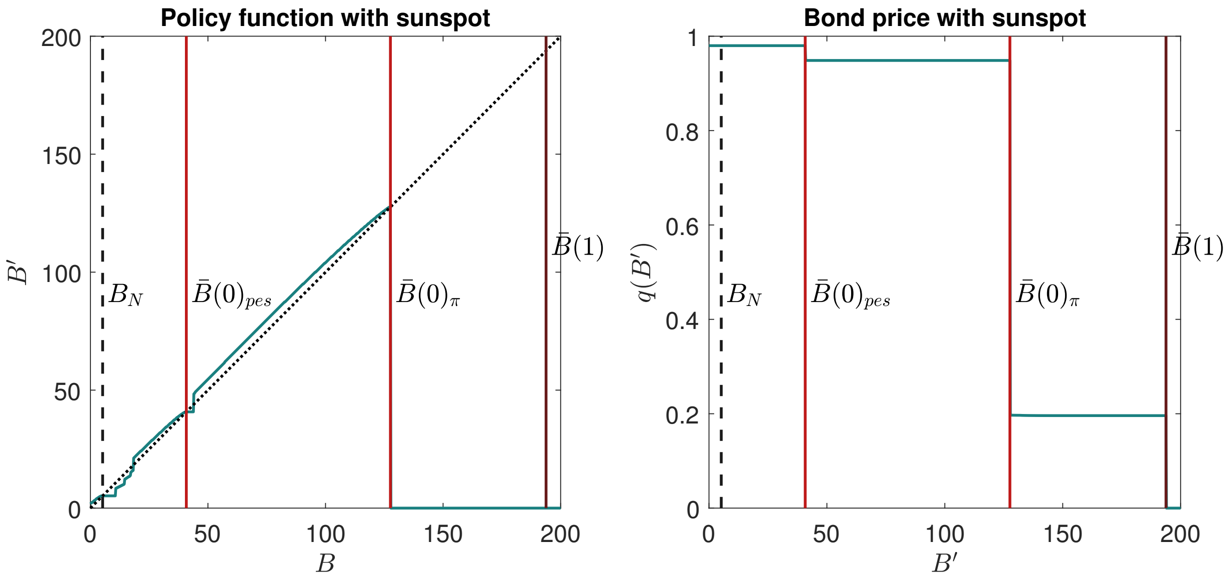

Figure 7: $\delta = 1.0$, $A = 0.9$, $p = 0.2$ with sunspot

and shifts in thresholds are now substantially different from Figure 6, there is a subtle change in the policy function. As for the case of long-term debt, there is optimal deleveraging for a range of debt above B_N. But now we also have optimal deleveraging for a very small range of debt just above $\bar{B}(0)_{pes}$. A key insight can be learnt from the right panel of the graph. Note that, once debt rises above B_N, investors keep lending at risk-free rate, even if it is understood that a switch in expectations may end up igniting a slow-moving debt crisis. There is no price effect in trespassing the threshold. The government's deleveraging decision only reflects the prospective loss of welfare (the 'cliff effect'). Conversely, although the cliff effect is absent at the threshold $\bar{B}(0)_{pes}$, the full pass-through of changes in market interest rates (about 3.4%) on government borrowing costs takes only one period. The aggravation of cost is a good enough incentive for the government to pursue some deleveraging for a stock of liabilities just above $\bar{B}(0)_{pes}$.

6 Multiplicity in the debt-limit model

In this section, we turn to economies in which the government is willing to exhaust all possibilities of adjustment before repudiating the debt and fall in a low-output financial autarky situation. Below we will show how this economy can be derived as a variant of our baseline, by positing that high output costs of defaulting compromise the government capacity to sustain essential fiscal spending.

To introduce the new framework, we find it useful to assume a given path of maximum adjustment in the primary surplus, and rewrite the default condition (3) for short-term debt as follows:

$$\theta y(a,z) - \bar{g} + \max_{B'}\{q(B',s)B'\} < B$$

Starting from this condition, the debt-limit default framework features similar dynamics to our baseline model. Debt tolerance thresholds may vary with investors' view on solvency, and it affects debt paths and policy function.

6.1 Debt thresholds and crises with short-term debt

In the debt-limit framework, the debt tolerance thresholds are pinned down by the maximum adjustment in primary surpluses the government is willing/able to generate. Yet they are not unique: they may be shifting in response to the regime of investors' expectations.

6.1.1 The debt tolerance threshold in normal times $\bar{B}(1)$.

In normal times, the government budget constraint is

$$B = \theta\bar{y} - g + q(B',s)B'$$

Since, once the economy recovers, it never falls back to a recession again, the government's optimization problem is deterministic—in normal times, there is no reason to borrow or lend for consumption smoothing purposes. If no default has occurred in the past, the government will simply service its existing debt at the risk-free rate, paying $(1-\beta)B$ to investors each period, to satisfy the no-Ponzi condition.

Given θ, the government will not default if and only if

$$B \leq \frac{\theta\bar{y} - \bar{g}}{1-\beta} = \bar{B}(1)$$

where $\bar{g}$ is the critical expenditure level.

6.1.2 The debt tolerance threshold in a recession $\bar{B}(0)$.

In a recession, the government budget constraint reflects the decline in tax revenue due to the downturn in activity ($A < 1$):

$$B = \theta A\bar{y} - g + q(B', s)B'$$

In a **pessimistic world**, investors are only willing to buy bonds at the low risky price. Given the definition of the debt tolerance threshold, the maximum the government can borrow is capped by the stock of debt that the government can service if the economy recovers, that is, $\max\{q(B', s)B'\} = \beta p\bar{B}(1)$. Now, to avoid default, the current debt must satisfy:

$$B < \theta A\bar{y} - \bar{g} + \beta p\bar{B}(1)$$

In other words, the government will not default if and only if

$$B \leq \theta A\bar{y} - \bar{g} + \beta p\bar{B}(1) = \bar{B}(0)_{pes}, \tag{12}$$

an expression that gives us the current debt tolerance threshold $\bar{B}(0)_{pes}$.

In an **optimistic world**, we need to examine two possible debt issuance strategies for the government. One consists of issuing a lot of debt, at a low, risky price—essentially this is the same strategy as described above, and is therefore associated to the same debt threshold (12). The other one consists of keeping new issuance in check, so to ensure that debt remains safe. This can be dubbed as a "low-risk low-debt" issuance strategy. By using the same steps above, we can derive the maximum sustainable debt conditional on the safe-debt strategy as:

$$B \leq \frac{\theta A\bar{y} - \bar{g}}{1 - \beta}$$

Thus, $\bar{B}(0)_{opt}$ can be characterized as follows:

$$\bar{B}(0)_{opt} = \max\left\{\frac{\theta A\bar{y} - \bar{g}}{1-\beta}, \bar{B}(0)_{pes}\right\}$$

Which strategy gives the government higher revenue in an optimistic world depends on parameters. If all government debt is short-term, we find that $\bar{B}(0)_{opt} > \bar{B}(0)_{pes}$, and thus a safe-debt strategy makes the government better off.

A notable implication is that, with short-term debt, the possible equilibria in a debt-limit framework are similar to the ones depicted in the three panels of Figure 2. The same analysis also applies: crises can be slow-moving, for intermediate level of debt, but can be fast, once the initial stock of debt is high enough (obviously the thresholds will be quite different in a debt-limit framework). However, in the next section, we will show that the crises in two frameworks may be quite different when debt is long-term.

6.2 Sustainability and crises

To study the debt-limit model, as already mentioned, we use a variant of our baseline. Instead of assuming (11), we posit that the government suffers a utility cost Γ if it cuts spending below $\bar{g}$. The objective function is as follows

$$u(c,g) = \log(c) + \mathbb{1}_{g>\bar{g}}\big(\gamma\log(g-\bar{g}+\epsilon)\big) - (1-\mathbb{1}_{g>\bar{g}}) * \Gamma,$$

where $\mathbb{1}_{g>\bar{g}}$ is an indicator function equal to 0 if spending falls below is critical value, and we assume an arbitrary small ϵ to ensure that $u(c,g)$ is bounded below when $g \to \bar{g}$. This is the key implication: if the output cost of defaulting $1-Z$ is large enough to bring spending below the critical level $\bar{g}$, and Γ is large enough, the value of repaying will never be below that of defaulting. Intuitively, the government understands that, conditional on defaulting, it will have insufficient fiscal capacity. Yet, as shown below, crises are possible, depending on the initial conditions, the persistence of recessions and investors' expectations.

Using this new framework, we now set $Z = 0.8$, $\theta = 0.35$, $\bar{g} = 30$ such that government spending falls below the critical level $\bar{g}$ upon a default.[13] For other

[13] We observe that the initial recessionary state can be quite adverse, i.e., A can be so low that

parameters except for maturity indicator δ, we adopt the same values as in the baseline of Table 1. To save space, we concentrate on consistent debt paths before the economy recovers.[14] We will show that, relative to our results so far, long-term debt tends to rule out "fast" debt crises more easily, but remains ineffective in ruling out "slow-moving" debt crises.

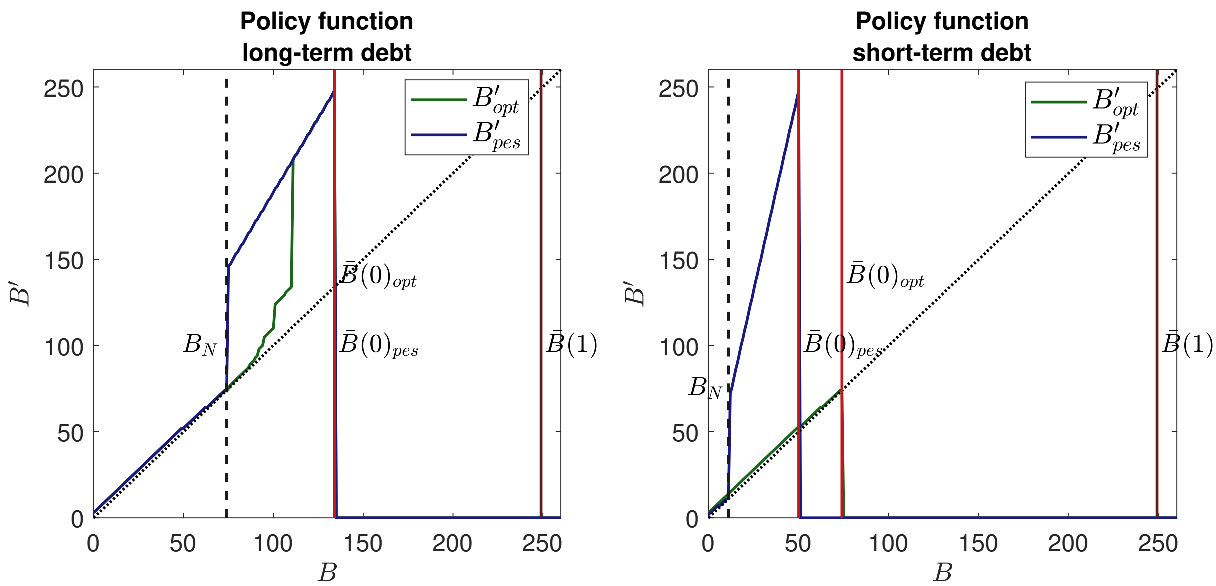

Figure 8: Policy functions where $\delta = 0.2$ (left) and $\delta = 1.0$ (right)

The main results from our exercise are shown in the two panels of Figure 8, which depicts policy functions with long-term bonds (left panel) and one-period bonds (right panel). Each panel illustrates both the optimistic and the pessimistic worlds.

The right panel of Figure 8 depicts the policy functions with one-period bonds. The debt dynamics are very similar to Figure 5 but the debt threshold $\bar{B}(0)_{opt}$ is much lower. In an optimistic world, the government accumulates debt over time to smooth consumption till it reaches $\bar{B}(0)_{opt}$. In a pessimistic world, the government issues safe debt at a slow pace in the region between 0 and B_N; it starts to accumulate risky debt at a fast pace in the region between B_N and $\bar{B}(0)_{pes}$. Fast, rollover crises can nonetheless occur for debt levels between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$.

The debt dynamics shown by the left panel in Figure 8 are quite different. The equilibrium is unique for a low level of debt (in the region between 0 and 74) and for

the government cannot finance the critical level of spending $\bar{g}$ without borrowing. In other words, $A\theta\bar{y} < \bar{g}$. We discuss this case in online Appendix D.

[14] We leave a more general analysis of sunspots to online Appendix E.

a high level of debt (in the region above 110). Multiplicity exists for intermediate levels of debt (in the region between 74 and 110). With long-term debt, $\bar{B}(0)_{pes}$ coincides with $\bar{B}(0)_{opt}$: "fast" debt crises are no longer possible.

This result suggests that debt maturity is much more consequential in the debt-limit than in the baseline model. As discussed below, in our calibration, we find that "fast" debt crises are ruled out in the debt-limit version of our model for any δ below 0.57, corresponding to a debt maturity of seven quarters—for any longer debt maturity, "none" and "slow" are the only possible outcomes in debt-limit framework.

7 What determines a debtor's resilience to self-fulfilling crises?

In this subsection we study how an economy may/may not be vulnerable to debt crises, depending on the maturity of its debt and economic conditions, that depth and expected persistence of a downturn. To so so, we carry out sensitivity analysis comparing our baseline with the debt-limit framework, and focus on conditions under which fast debt crises can be ruled out altogether.

For our baseline (strategic default) model, in Figure 9 we plot debt tolerance thresholds in a recession as we vary debt maturity (left panel) and the probability of recovery (right panel).

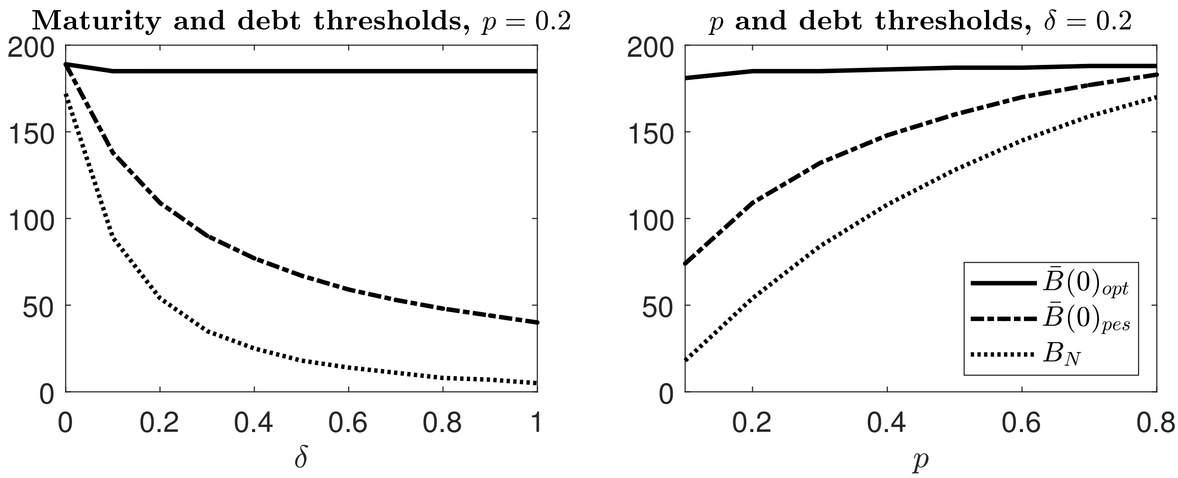

Figure 9: Debt thresholds in the baseline model given $A = 0.9$

Starting from the left panel of Figure 9, we first note that $\bar{B}(0)_{opt}$ is insensitive to debt maturity, but for extremely small values of δ ($\delta \to 0$), corresponding to very long maturities. As long as investors remain optimistic, the government can borrow at risk-free rate. Long-term debt and short-term debt are basically equivalent—a small effect can be detected only at extreme maturities, reflecting a lower incidence of debt rollover on the gross financing need.

In contrast, both B_N and $\bar{B}(0)_{pes}$ decrease sharply with δ, that is, they increase with a longer debt maturity of debt. To see why, consider the net bond revenue in a pessimistic world, $\beta p(B' - (1-\delta)B) - \kappa B$, where $\beta p B'$, $\beta p(1-\delta)B$ and κB denote, respectively, revenue from newly issued bonds, the value of the outstanding stock of bonds, and interest payment to investors. Maturity has opposite effects on net bond revenue. As the maturity of bonds becomes longer ($\delta \downarrow$), the value of the outstanding stock of bonds $\beta p(1-\delta)B$ rises but the interest payments due in the period κB fall. The first effect decreases, while the second effect increases the net bond revenue. But rearranging the net revenue equation as follows: $\beta p B' - [1 - \beta(1-p)(1-\delta)]B$, it is apparent that the second effect always dominates the first one: a fall in δ unambiguously increases net debt revenue—explaining why $\bar{B}(0)_{pes}$ and B_N are larger as the debt maturity becomes longer. One could note that, when investors hold a pessimistic view of the government, an official swap of short-term bonds for long-term bonds may improve the debt tolerance threshold of a country (a point discussed is detailed in Corsetti et al. (2017)).

Observe that, in the figure, the "fast" crisis zone, the distance between $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$, becomes wider, the shorter debt maturity is. On the contrary, the "slow-moving" crisis zone, the distance between B_N and $\bar{B}(0)_{pes}$, remains approximately unchanged as maturity shortens—B_N and $\bar{B}(0)_{pes}$ basically fall at a similar pace.

The right panel of Figure 9 shows that the probability of recovery p does not have much of an effect on $\bar{B}(0)_{opt}$, while it has a significant impact on both B_N and $\bar{B}(0)_{pes}$. The net bond revenue in an optimistic world, $\beta(B' - (1-\delta)B) - \kappa B$, does not vary with p, while the net bond revenue in a pessimistic world, $\beta p(B' - (1-\delta)B) - \kappa B$, is unambiguously increasing in p. A higher probability of recovery p significantly narrows the "fast" crisis zone. It also narrows, but to a lesser extent, the "slow-moving" crisis zone.

We repeat this sensitivity analysis for the debt-limit framework—results are shown in Figure 10. There is at least one significant difference relative to our baseline. Focusing on the left panel in Figure 10, note that the debt threshold $\bar{B}(0)_{opt}$ is insensitive to debt maturity only for a δ higher than 0.57, that is, for relatively short maturities. With short-term debt, a low-debt safe-debt issuance strategy yields higher revenue than high-debt risky-debt issuance strategy—it is rationale for the government to remain on the good side of the Laffer curve. As explained in our comments to the previous figure, intuitively, the revenue from a low-risk issuance strategy is not affected by δ because investors lend at risk-free rate. However, once δ becomes smaller than 0.57, i.e., once debt maturity becomes sufficiently long, the government rationally switches to a high debt, risky-debt issuance strategy even in the optimistic world. Different from the left panel of Figure 9, $\bar{B}(0)_{opt}$ is now the same as $\bar{B}(0)_{pes}$, both increasing in debt maturity (lower δ). The remarkable implication is that, with longer maturities, the "fast" crises zone no longer exists.

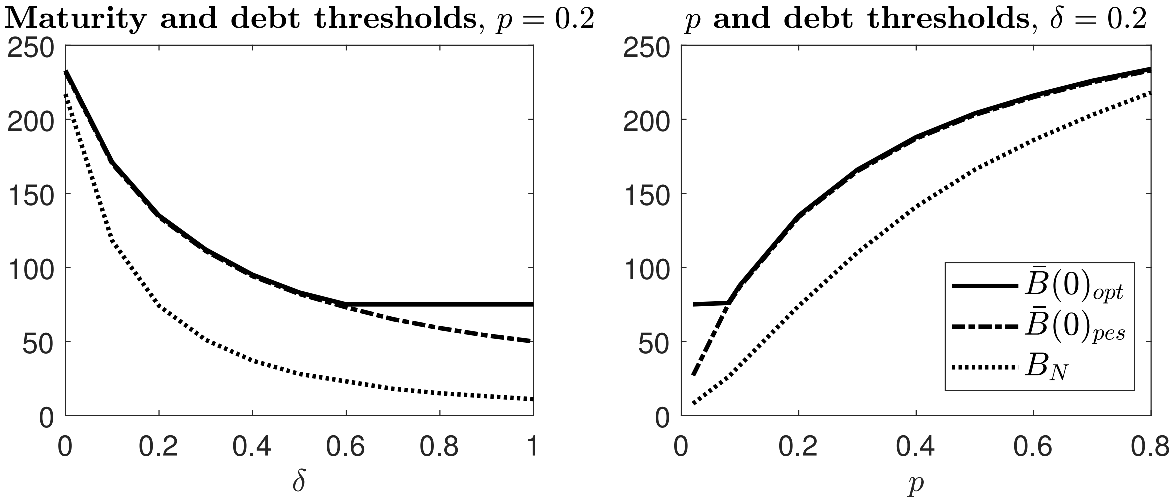

Figure 10: Debt thresholds in the debt-limit framework given $A = 0.9$

A similar picture is provided by the right panel of Figure 10, which plots debt thresholds against the probability of recovery p. Different from the corresponding panel in Figure 9, $\bar{B}(0)_{opt}$ now rises substantially with a higher p, and coincides with $\bar{B}(0)_{pes}$ for any p larger than 0.08. Only for a very low probability of recovery, the low-debt safe-debt issuance strategy dominates the high-debt risky-debt issuance strategy in the optimistic world, causing $\bar{B}(0)_{opt}$ to diverge from $\bar{B}(0)_{pes}$, and to remain insensitive to p. For any non-negligible probability of recovery (p larger than 0.08), risky-debt strategy generates higher revenue and the "fast" crises zone disappears.[15]

[15]"Fast" debt crises can be ruled out in baseline as well, but the parameter restrictions are more

Overall, our sensitivity analysis highlights a striking difference between the baseline model and the debt-limit framework. For intermediate debt maturities and non-negligible probability of recovery, fast (rollover) crises are still possible in the former, but not in the latter. The reason is that, in debt-limit framework, the government takes advantage of long debt maturities and good recovery prospect to raise revenue by issuing (more) risky debt—de facto putting the economy in a slow-moving debt crisis mode.

8 Conclusion

The literature has long emphasized that, once the stock of their debt is sufficiently high, the equilibrium is no longer unique and countries are vulnerable to disruptive self-fulfilling crises. As the Covid-19 pandemic is causing widespread economic crises across the globe, it is unavoidable that debt stocks rise virtually everywhere, potentially undermining stability in the bond markets in advanced countries and raising issues in which instruments are available to keep these markets in a "good equilibrium."

Market instability can take different forms: understanding market dynamics during a crisis is of fundamental importance for policy assessment and design. Seminal contributions to the literature have shown how at any point in time a switch in market expectations from a good to a bad equilibrium may result in higher borrowing costs causing debt accumulation—the crisis then develops over time, from a combination of an unsustainable build-up of debt and fundamental stress, as in Calvo (1988) and Lorenzoni and Werning (2019). Other seminal works emphasize that the switch causes a country to lose market access: a rollover crisis forces a sudden default, as in Cole and Kehoe (2000).

In this paper, we have stressed both types crises may occur in the same dynamic Calvo (1988) setting. When the regime of investors' expectations turns from optimistic to pessimistic, higher costs of debt either reduce the social utility of not defaulting, or force the government to issue a high volume of risky debt and gamble on future recoveries. In either case, the debt tolerance threshold of the government—the

stringent. That is, the country is in a deep recession, the probability of recovery is high, and debt maturity is sufficiently long. These results are shown in online Appendix F.

debt level above which default becomes the dominant action—falls with the switch in investors' expectations. At this switch, depending on the initial level of debt, investors may anticipate that the government will not be willing/able to undertake the required adjustment to sustain debt at any finite equilibrium interest rates. At sufficiently high levels of debt, countries able to borrow at the risk-free rate may suddenly lose market access and, in the absence of external official support, default. We also show that this is true both when the government has enough fiscal spending capacity after defaulting, so that it can act strategically, as well as when the costs of the default compromise the spending capacity of the government, so that this is forced to act at the "debt limit"—essentially generating the maximum primary surplus that is economically and politically sustainable.

By characterizing a sunspot equilibrium, we are able to revisit debt dynamics and deleveraging under the threat of a rollover crisis. This is an important question that may dominate fiscal policy in the post-Covid crisis, high debt regime. As in the existing literature, we find that a forward-looking benevolent government reduces debt during recessions, motivated by the prospective loss of welfare in a belief-driven crisis and high costs of borrowing. Different from the existing literature, however, in our framework, this type of deleveraging is optimal only for a relatively small range of debt, at low levels close to the debt threshold at which the country becomes exposed to slow-moving debt crisis. On the contrary, deleveraging is not generally optimal around the higher debt threshold at which the country becomes exposed to fast, rollover crises. This result suggests that debt level may persist for a long time in the region where countries are exposed to the threat of a belief-driven debt run, as this threat is not generally enough even for forward-looking benevolent governments to embrace precautionary fiscal policy of risk reduction. In light of contagion effects undermining stability at global level, there is a strong argument for international fiscal compacts and institutional liquidity provision, to foster a smooth recovery.

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