The Economy of Risk in Insurance in Insurance

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Agenda

1	Definition of risk and risk measures
2	A simple example of risk to price
3	Pricing the risk in a portfolio
4	Conclusions

What is Risk? Dictionary Definitions

- The New Shorter Oxford English Dictionary:
  1. *Danger; (exposure to) the possibility of loss, injury or other circumstance*
  2. *A chance or possibility of commercial loss …*
  3. *A person considered a liability or danger; a person exposed to risk*

- Webster's College Dictionary (insurance):
  1. *The chance of loss*
  2. *The degree of probability of loss*
  3. *The amount of possible loss to the insuring company*
  4. *A person or thing with reference to the risk involved in providing insurance*
  5. *The type of loss that a policy covers, as fire, storm, etc.*

What is Risk? Encyclopedia and Philosophy

- Wikipedia:

  Risk *is a concept that denotes the precise probability of specific eventualities. Technically, the notion of risk is independent from the notion of value and, as such, eventualities may have both beneficial and adverse consequences. However, in general usage the convention is to focus only on potential negative impact to some characteristic of value that may arise from a future event.*

- The French philosopher *Etienne de Condillac* (1714-1780):

  The chance of incurring a bad outcome, coupled, with the hope, if we escape it, to achieve a good one.

- We are willing to take the risk to have the chance of obtaining a positive outcome

- During the XVIIIth century appears the notion that social institutions should protect people against risk. This contributed to the development of life insurance that was not really acceptable by religion at the time (and still is forbidden in Islam)

Risk Management and Efficient Market Theory (1/2)

- Most Risk Management (RM) methods rely on the Efficient Market Hypothesis (EMH), but this assumption implies that RM does not add value to shareholders*
- We can see it easily, when we look at the Risk Neutral Value (RNV) of the firm:
  - The value of the firm *without* RM = RNV(future earnings)
  - The value of the firm *with* RM = RNV(future earnings + net transfer from RM instruments)
  - The EMH implies additivity of RNV and that the RNV(net transfer from RM instruments)=0 (It is how their prices are computed)
  - Thus: *value of the firm without RM = value of the firm with RM*

*) Inspired ba a presentation from Jean-Charles Rochet at PRMIA sept. 2013

Risk Management and Efficient Market Theory (2/2)

- RM would only be useful to optimize taxes or to protect managers at the expenses of the shareholders as RM implies also costs
- This model is only valid when we assume *complete markets* and *no transaction costs* (frictions), which is never the case in real life
- Without these assumptions, RM becomes a vital function in a financial institution
- We do believe that RM increases the value of a firm to shareholders and thus it makes sense to talk about "The Economy of Risk in Insurance" and see how it can be used to improve our business

Risk and Uncertainty, Two Related but Different Concepts

- Risk and uncertainty are often used indistinctly, but in economics the tradition wants to distinguish between the two concepts.
- The American economist Frank H. Knight (1885-1972) was the first to propose *a clear distinction between risk and uncertainty* in his famous dissertation *Risk, Uncertainty and Profit* (1921), which is now commonly used in economics.
- He defines “Risk" as the randomness with knowable probabilities (measurable uncertainty), while “Uncertainty" is the randomness with unknowable probabilities (unmeasurable uncertainty).
- In Donald Rumsfeld’s terminology risk is the “*known unknowns*”, while randomness is the “*unknown unknowns*”.

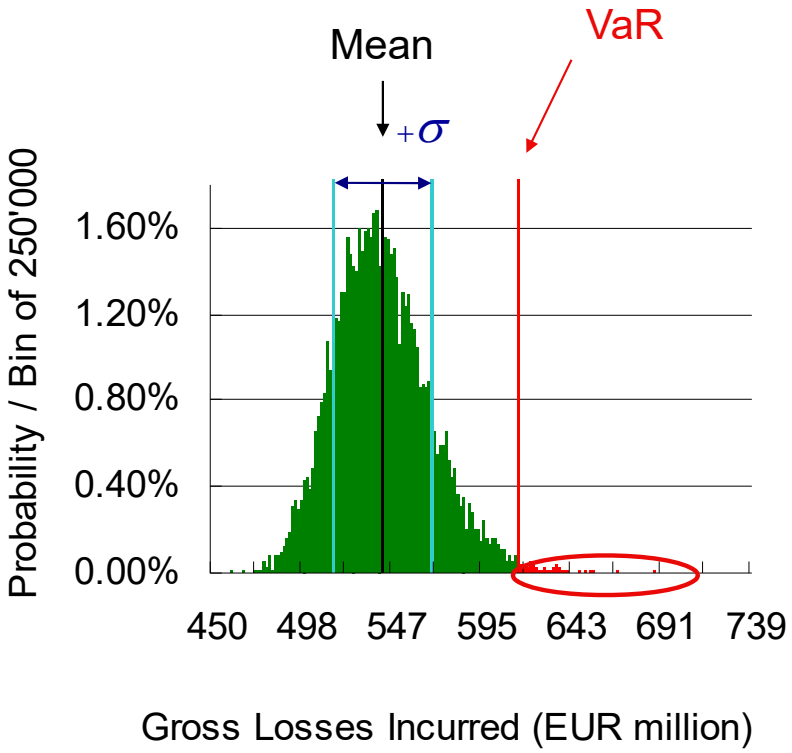
The Concept of Risk in Insurance

- In insurance risk is defined as: *the variation from the expected outcome over time.*
- We all have expectations about results – but the actual outcome is uncertain.
- Risk quantifies the *uncertainty* of the future outcome of a current decision or situation.
- Insurance is the transfer of risk from an individual to a group (company).
- The premium should reflect the risk assumed by the company and the diversification of the insurer's portfolio.
- In a model, the possible outcomes can be adequately described by a *probability distribution*.

Risk and Risk Measures

- Risk can be measured in terms of probability distributions, however, it is some time useful to express it with one number. This number is called *risk measure*.
- We want a measure that can give us *a* risk in form of *a capital amount*.
- The risk measure should have the following properties *(coherence)*:
    1. Scalable (twice the risk should give a twice bigger measure),
    2. Ranks risks correctly (bigger risks get bigger measure),
    3. Allows for diversification (aggregated risks should have a lower measure),
    4. Translation invariance (proper treatment of riskless cash flows),
    5. Relevance (non-zero risks get non-zero risk measures).

Loss Model and Risk Measures



Standard Deviation
measures typical size of fluctuations

Value-at-Risk (VaR) $\mathrm{VaR}_\alpha(L) = \mathrm{Inf}\{l \in \mathbb{R}: P(L > l) \leq 1 - \alpha\}$
measures position of 99th percentile,
„happens once in a hundred years“

Expected Shortfall (ES) is the weighted average VaR beyond the 99% threshold. It is also called TVaR (tail VaR): $\mathrm{ES}_\alpha(L) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_u du$

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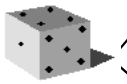
A Simple Example of Pricing Risk

- Assume an insurance customer approaches a company with the aim to insure the following risk:
  - ✓ He must pay 10 EUR if he gets a six on a dice, and nothing otherwise.
  - ✓ He must throw the dice 6 times.

- We will answer two questions:
  1. What is the price for such a risk, independently of any other liability the insurer has?
  2. How would the price change if the insurer assumed many of the same risks?

Outcome of Throwing the Dice



First Throw

Out.	Prob.	Claim
6	1/6	10

Out.	Prob.	Claim
<6	5/6	0

Second Throw

Out.	Prob.	Claim
6	1/6	10

Out.	Prob.	Claim
<6	5/6	0

Out.	Prob.	Claim
6	1/6	10

Out.	Prob.	Claim
<6	5/6	0

Total	
Probability	Claims
$\frac{1}{6} * \frac{1}{6} = \frac{1}{36}$	20
5/36	10
5/36	10
25/36	0

And so on …

The Probability Model

- Consider a fair game, when the dice is unbiased and it is equally probable to get one of the six faces of the dice
- Let *X* be a *Bernoulli random variable* (rv) defined on a probability space $(\Omega, \mathbb{A}, \mathbb{P})$ representing the loss obtained when throwing the unbiased dice, i.e when obtaining a "6"

$$X = \begin{cases} 1 & \text{with probability p} = 1/6 \\ 0 & \text{with probability } 1 - \text{p} \end{cases}$$

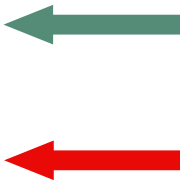
- Let X_i, *i=1 … n*, be a *n*-sample with parent rv *X*, corresponding to the sequence of throws when playing n times (independent games). In our example, *n=6*.The number of losses after *n* games is modeled by $S_n = \sum_{i=1}^{n} X_i$, which is distributed according to a *binomial distribution*

$$\mathbb{P}(S_n = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

- With $\mathbb{E}(S_n) = np$ and $\sigma^2(S_n) = np(1-p)$

The Cumulative Distribution of Our Example

Loss Amount	Loss Probability	Cumulative Probability
0	33.490%	33.490%
10	40.188%	73.678%
20	20.094%	93.771%
30	5.358%	99.130%
40	0.804%	99.934%
50	0.064%	99.998%
60	0.002%	100.000%



Expected Loss = 10 EUR

Value-at-Risk (99%) = 30 EUR

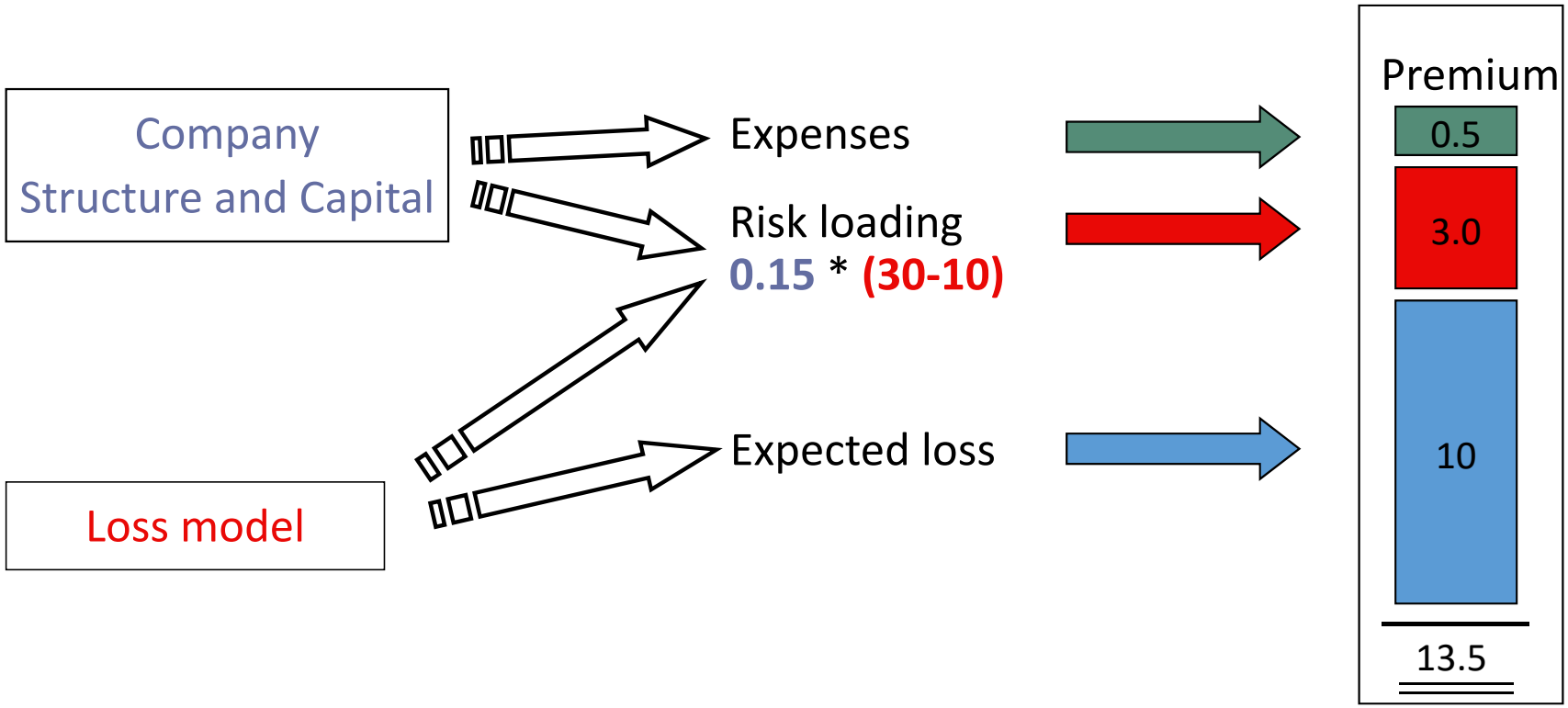
What is the Correct Price?

- Pricing the risk at the *expected loss* plus expenses means running the risk of losing more
- Here there is a 26% chance to pay more than the expected 10 EUR
- The risk is to have a claim that exceeds the expected loss by far
- We define the risk as the *unexpected* loss
- The role of an insurer is to guarantee payment of the loss even if it is above expectation
- Thus, the need to put up capital to cover this risk *(Risk-Adjusted Capital, RAC)*

Determining the Capital to Cover the Risk

- In order to quantify the risk, we need to define up to which probability the insurer is willing to guarantee payment
- This is the *confidence threshold* at which the company wants/needs (because of solvency regulation) to operate (let us choose here the “1 over 100 event”)
- Such a confidence threshold corresponds in our case to a claim of 30 EUR
- Thus, the capital would be 20 EUR (the 99% claim minus expected claim)
- Providing *capital has a cost* – investors want a Return on Investment
- Let us assume a cost of capital of 15% before tax and *company expenses* of 0.5% of the expected loss

Computing the Premium



Diversification: Insuring Many Independent Risks Together

- Such a price is uneconomical: It is *too expensive*
- Assume that the insurer takes on *not only* the risk of *one* policyholder *but many*
- Each policyholder insures the risk that he has to pay EUR 10 in each case a 6 appears on a dice at 6 throws
- Many risks will constitute a *portfolio of risks* now
- How will the premium change due to diversification?
- Remember: The expected loss per policy was EUR 10, expense EUR 0.5 and the risk loading EUR 3 for one policy seen in isolation

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Influence of Diversification on the Premium

Number of Policies	Cost of Capital
1	3.0000
5	1.5000
10	1.0500
50	0.4500
100	0.3300
1000	0.1020
10000	0.0320

As can be seen, if the risks diversify, the risk loading per policy is *strongly reduced*, the more policies are part of an insurer's portfolio.

Limits to Diversification

- We have in the previous slide assumed that *all the policies are independent*
- Let us now introduce a *less restrictive assumption* and assume that not all risk are diversifiable
- Assume that the policyholders throw the dice all in the same casino and that with a given probability p, they will have a crooked croupier. In that case, they will *all (or a high proportion of them)* lose EUR 60, i.e. each throw of a dice will show a 6 with another probability $q>>p$
- Such a biased probability introduces a risk that is common to all policies and thus cannot be diversified away by taking more of those policies in the portfolio. We call this the *non-diversifiable risk*.

Effect of non-diversifiable risks with VaR*

Number of Policies	Risk Premium based on VaR* 0%	0.1%	1%	5%	10%
1	2.964	2.997	4.469	4.346	5.693
5	1.425	1.497	2.070	3.450	3.900
10	1.007	1.047	1.770	3.300	3.450
50	0.449	0.477	1.410	3.060	3.030
100	0.318	0.327	1.605	3.000	2.940
1000	0.101	0.101	2.549	2.900	2.775
10000	0.032	0.029	2.837	2.865	2.724
Expected Loss	10.00	10.02	10.20	11.00	12.00

Diversification is significantly reduced if there are underlying risk factors affecting all policies simultaneously (e.g. a crooked croupier).

*) The biased dice has a probability of hitting a "6" of q=0.5

Effect of non-diversifiable risks on the TVaR*

Number of Policies	Risk Premium based on TVaR* 0%	0.1%	1%	5%	10%
1	3.226	3.232	4.711	4.755	5.899
5	1.644	1.707	2.956	3.823	4.146
10	1.164	1.266	2.973	3.578	3.665
50	0.510	0.760	2.970	3.196	3.141
100	0.372	0.596	2.970	3.098	3.020
1000	0.116	0.396	2.970	2.931	2.802
10000	0.037	0.323	2.970	2.876	2.732
Expected Loss	10.00	10.02	10.20	11.00	12.00

TVaR is more sensitive to the enhanced risks and shows a significant loss of diversification even with ~p=0.1% .

*) The biased dice has a probability of hitting a "6" of q=0.5

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Conclusion

- Risk in insurance is the deviation of the expectation
- The concept of Risk-Adjusted Capital (RAC) is central for understanding the value creation of an insurance company
- The definition of RAC depends on the risk measure used and the risk appetite of the company
- An insurance contract is only economically viable if taken in a portfolio of diversifying risks
- Pricing risk means modeling the entire portfolio
- A sound capital allocation methodology allows to price the risk of an insurance contract to provide the appropriate Return on Equity